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The complexity of problems in wireless communication

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Abstract Ad hoc networks become increasingly important in our life, for their advantages without relying on existing infrastructures and for their ability to be fast implemented, especially in the aspects of rescue after disasters and military. However, since every node in an ad hoc network can move freely, we are confronted with many new problems when compared with cellular networks and WiFi, such as the change of connectivity between nodes and signal interference and blockage by obstacles. Thus, it is important to understand solutions and complexities of various programming problems in ad hoc networks. In this paper, based on an existing mobility model for ad hoc networks, we study solutions and complexities of a series of problems proposed by Greenlaw, Kantabutra, and Longani, including the multiusers simultaneous communication problem (MUSCP), the longer communication problem (LCP), the obstacle removal problem (ORP) and the user communication, limited number of sources problem (UCLNSP). For MUSCP and LCP, we provide efficient algorithms to solve them and prove that they are P problems. On the other hand, for ORP and UCLNSP, by applying reduction from the set covering decision problem, we prove that they are NP-complete, and thus, they are intractable, unless $P = NP$.

Keywords Ad hoc network · Algorithm · Complexity · Wireless mobile communication

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1 Introduction

In recent years, wireless ad hoc networks have been drawing increasing attention due to their potential applications in civil and military domains [1,2]. Without any fixed preinstalled infrastructure, all nodes in wireless ad hoc networks can move arbitrarily. Each of them works both as a router and as a host. They dynamically establish routing among themselves to form a temporary network [3]. Compared with infrastructured wireless networks, such as cellular networks and WiFi, wireless ad hoc networks have the advantage of being fast deployed as well as low cost. Thus, they are particularly useful in disaster areas and battlefields.

However, when deploying a wireless ad hoc network, we are confronted with many different problems compared with infrastructured wireless networks, such as how many access points to be deployed, how to move them and how to route among them, and meanwhile satisfying various constraints such as duration of communication or limitation of power. In addition, when it is in the aspects of military and disaster recovery, if we cannot finish the deployment of the communication system meeting the requirement, then both our military and the victims of disasters will suffer from a deadly blow. Motivated by this, there is an urgent desire to understanding the solutions and complexities of related problems in wireless ad hoc networks.

In this paper, we study the computational complexity of a series of problems in wireless communication within the mobility model proposed by Greenlaw et al. [1], which is based on a two dimensional grid and incorporates elements of users, access points, and obstacles. This model strikes a balance between simplicity and utility. Thus, it is appropriate in investigating the complexity of problems in ad hoc networks. Greenlaw et al. [1] proposed the following five open problems: the multiusers simultaneous communication problem (MUSCP), the longer communication problem (LCP), the obstacle removal problem (ORP), the user communication, limited number of sources problem (UCLNSP), and the access point placement problem (APPP). We solve them all but the last one completely in this paper. For the first two problems, we give efficient algorithms to solve them, and prove that they are both in P. For the next two problems, we prove that they are both NP-complete.

This article is organized as follows: Sect. 2 presents related works, and Sect. 3 presents the mobility model we use in this work. Section 4 investigates two problems in P and presents their computational complexity. Section 5 investigates the other two problems in NP and gives the proof of their NP-completeness. Finally, we conclude with future work in Sect. 6.

2 Related works

Works related to our paper can be divided by three topics: wireless mobility models, complexity of other problems in wireless communication, other important problems of wireless networks.

There have been a number of works about the mobility model of wireless ad hoc networks. As we introduced before, Greenlaw et al. [1] proposed a mobility model of wireless ad hoc networks based on a two dimensional grid, which incorporates elements of users, access points, and obstacles. Ahmed et al. [4] proposed an environment-aware mobility model for wireless ad hoc networks. Real world features such as obstacles and doorways are incorporated in this model to provide a movement pattern which resembles more like what happens in real world. Zarifneshat and Khadivi [5] proposed a more realistic mobility model for ad hoc networks based on random mobility models and Levy walk mobility model [6]. Instead of sudden speed change, they introduced accelerated movement in their model. For example, when mobile objects are going to change their direction, they decelerate before reaching a direction change point, change their direction, and then accelerate from zero to its selected speed. Differently in our work, we focus on the computational complexity of related problems in wireless communication. By applying detailed patterns or probability distribution of nodes or users' movement, one can only obtain the average or empirical time cost of related problems rather than their theoretical computational complexity. Thus, we use the simple mobility model without assumption of nodes' mobility patterns of wireless ad hoc networks in [1].

In addition, there are also many works investigating complexity of other problems in wireless communication. Moscibroda and Wattenhofer [7] studied the scheduling complexity of connectivity, i.e., the minimal amount of time required until a connected structure can be scheduled. They proved that this problem can be solved in polynomial time. Andrews and Dinitz [8] studied the problem of maximizing the number of supported connections by choosing transmission powers for each connection under the SINR model. They showed this problem is NP-hard, and presented a number of approximation algorithms for the problem, which run in polynomial time. Koushanfar and colleagues [9] studied the problem of calculating exposure in wireless ad hoc sensor networks. They introduced an exposure-based coverage model, formally defined exposure and studied several of its properties, including the complexity of the problem. Combining computational geometry and graph-theoretic techniques, they developed an efficient and effective algorithm for minimal exposure paths for any given distribution and characteristics of sensor networks. Li et al. [10] focused on the reliable broadcast and multicast lifetime maximization problems in energy-constrained wireless ad hoc networks. In unreliable networks, they proved their NP-completeness by a reduction from a well-known minimum degree spanning tree problem. Then, they proposed a link quality-aware heuristic algorithm which builds broadcast tree to maximize the network lifetime, and provided its time complexity. Ren et al. [11] investigated the connected dominating set (CDS), which has been widely studied to form virtual backbones for designing the stable and highly efficient network architecture in wireless ad-hoc sensor networks. The authors focused on constructing the minimum CDS, which has been shown to be NP-hard. Then, they proposed an algorithm which first finds a prior CDS and then uses the minimum-weight spanning tree to optimize the result. Theoretical analysis and proofs for the time complexity were also provided.

There are some works focusing on other important problems in wireless networks, such as [12–18]. Kapoor and colleagues [12] studied the clustering algorithms, which play a very important role in the fast connection establishment of ad hoc networks. They described a communication model derived from Bluetooth, and proposed a two-stage distributed $O(n)$ randomized algorithm and a completely deterministic $O(n)$ distributed algorithm for clustering of a wireless ad hoc network on n nodes. Another important problem in wireless networks is the energy consumption. For example, transmission energy required for a wireless communication increases superlinearly with the communication distance. Thus, if communication can be postponed until the sender moves close to a target receiver, then the energy used can be greatly reduced. Chin and colleagues [13] focused on this problem. They developed a general map-based network and movement model to capture realistic nodal movement, and derived tight lower bound expected communication distances achievable by any postponement algorithm, as a function of the average nodal speed and the allowable postponement delay. Cerulli et al. [14] also focused on the problem of ensuring reliability of a wireless sensor network while maximizing its lifetime. Different with the traditional approach of solving the k-maximum lifetime problem (k-MLP), which asks that each target to be covered by at least k different sensors, they proposed an alternative strategy where sensors adapt their sensing radii in response to failures to restore feasibility only when needed, of which the problem is referred to as potential k-MLP (Pk-MLP). Then, they provided column generation exact algorithms for both the traditional approach and their variant, as well as a heuristic procedure for the coverage restoration phase. Ao et al. [15] investigated the connectivity of cooperative secondary network from a percolation-based perspective, and characterized its connectivity in terms of percolation threshold. Khabbazian et al. [16] investigated the problem of minimizing interference by assigning transmission radii of wireless nodes with given positions. They proposed a local algorithm which provided an upper bound on expected maximum interference. Rosati et al. [17,18] focused on the problem of routing in flying ad hoc networks. They proposed an extension of the optimized link-state routing protocol [19,20], and compared their performance by both media-access-control layer emulations and real-world experiments.

3 The model

In this section, we present the mobility model we use for studying wireless communication, which is proposed by Greenlaw et al. [1]. This mobility model is selected since it strikes a balance between simplicity and utility. For example, as discussed in Sect. 2, we focus on the theoretical computational complexity of related problems in wireless ad hoc networks. Thus, details such as the probability distribution of nodes' movement should be ignored as in [1]. On the other hand, elements of users, access points, and obstacles and their corresponding features, e.g., the speed and direction of nodes, are all incorporated in this model. Thus, it is useful enough to provide the insight into real situations.

On the whole, this model operates on a two-dimensional grid. There are a number of wireless access points on the gird, which are referred to as *sources*, and a series of users communicate with each others by using the sources. Each source has a circular coverage with its specific radius. In addition, there are also some obstacles on the grid, which may block off the communication between sources and users.

Specifically, this model can be represented as an eight-tuple $\mathcal{M} = (\mathcal{S}, \mathcal{D}, \mathcal{U}, \mathcal{L}, \mathcal{R}, \mathcal{V}, \mathcal{C}, \mathcal{O})$, in which $\mathcal{S}$ is the set of sources, $\mathcal{D}$ is the set of the directions corresponding to movement of sources and users, $\mathcal{U}$ is the set of users, $\mathcal{L}$ is the set of strings of directions in $\mathcal{D}$ which represent a step in a given direction or no movement of the corresponding user, $\mathcal{R}$ is similar with $\mathcal{L}$, but it represents the movement directions of sources, $\mathcal{V}$ is the set of velocities of sources, that is, the number of steps per unit time that each source can move, $\mathcal{C}$ is the collection of radius of the circular coverage of sources; and $\mathcal{O}$ is the set of obstacles.

In more detail, $\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$, where m is the number of sources. For each $i \in \{1, 2, \ldots, m\}$, s_i represents a source, and its initial location is (x_i, y_i). Since the model is established on the grid, we have $x_i, y_i \in \mathbb{N}$. The set of directions $\mathcal{D} = \{000, 001, 010, 101, 110\}$, where the five elements represent no move, east, west, south, and north, respectively. $\mathcal{U} = \{u_1, u_2, \ldots, u_p\}$, where p is the number of users. For each $i \in \{1, 2, \ldots, p\}$, u_i represents a user, and its initial location is $(x_i^u, y_i^u) \in \mathbb{N}^2$. $\mathcal{L} = \{l_1, l_2, \ldots, l_p\}$, where l_j, a string composed of directions in $\mathcal{D}$, represents the movement direction of the user u_j at each time unit. In addition, the length of each l_j is τ, which is the duration of the model. Similarly, $\mathcal{R} = \{r_1, r_2, \ldots, r_m\}$ represents the movement direction of sources. $\mathcal{V} = \{v_1, v_2, \ldots, v_m\}$, where v_i is the velocity of the source s_i. Since the model is established on the grid, we also have $v_i \in \mathbb{N}$, indicating that the source s_i can take v_i steps per unit time. $\mathcal{C} = \{c_1, c_2, \ldots, c_m\}$ represents the radius of the circular coverage of sources. The coverage of the source s_i is a circle whose centre is at (x_i, y_i) and radius is c_i. Its representation in a grid is defined as follows:

Definition 3.1 (*Coverage Representation*) A coverage of radius c located at a fixed grid point is represented by the set of lattice points within the coverage and on its boundary.

$\mathcal{O} = \{o_1, o_2, \ldots, o_d\}$ is the set of obstacles. Similarly, the representation of the obstacle is defined using the set of lattice points as follows:

Definition 3.2 (*Obstacle Representation*) An obstacle in a grid is represented by the set of lattice points within the obstacle and on its boundary.

In the original mobility model of [1], each obstacle is a rectangle in the plane represented by a four-dimensional vector $(x_1, y_1, x_2, y_2) \in \mathbb{N}^4$, where $x_1 < x_2$ and $y_1 < y_2$. Its coverage area is the rectangle whose vertices are (x_1, y_1), (x_1, y_2), (x_2, y_1) and (x_2, y_2). Without loss of generality, we further assume that each obstacle is not adjacent with any other obstacle in our paper, that is, any pairs of obstacles do not intersect at all. It can be realized by merging any adjacent obstacles as a single one obstacle which is composed of finitely many rectangles in our setting. In addition, for all coordinates x_1, y_1, we have $x_1, y_1 < o_{\max}$, where $o_{\max}$ is a constant in $\mathbb{N}$ representing the boundary of the grid plane.

Next, we will make some discussion about the mobility model. This model is established on a series of simplifying assumptions. First about the grid, it makes all coordinates limited to a discrete grid. In addition, all users can only move one step per unit time, and the source s_i can move a fixed number of steps, which is determined by v_i in $\mathcal{V}$, since the walking speed of humans does not appear to be much different from each other, but the source might be a hummer or an elephant, whose speed is very different. On the other hand, it also assumes that all the walks have the same length. Of course, users and sources do not always travel at a constant velocity. However, by using no movement bit string 000 from $\mathcal{D}$ interleaved with bit string of other directions, shorter walks can be pad out. Similarly, by using bit string of two different directions which occur alternatively, combinational directions such as north–east, north–west can be approximated. In addition, by shortening the time unit and grid interval, the model can be more close to the reality. As for communication between sources and users, this model assumes that two sources can communicate if and only if their overlapping-coverage area is not completely contained inside obstacles, in which the overlapping coverage area is defined as follows:

Definition 3.3 (*Overlapping Coverage Area*) Let s and s' be a pair of coverage or obstacles in a grid. We say that s overlaps s' if and only if $|s \cap s'| \geq 2$. We call $s \cap s'$ an overlapping coverage area.

Under this condition, the case of tangency of their coverage areas can be excluded. On the other hand, a source s_i and a user u_j can communicate with each other if and only if u_j is within the coverage area of s_i and the line between their locations (x_i, y_i) and (x_j^u, y_j^u) does not intersect with any obstacle from $\mathcal{O}$. Users do not communicate with each other directly, and they communicate through a series of sources. Two users u and v can communicate with each other if and only if there exist a series of sources $s_1, s_2, \ldots, s_k$, $k \geq 1$, in which s_1 can communicate with u, s_k can communicate with v, and s_{n-1} can communicate with s_n for all $n \geq 2$.

Figure 1 shows an example of the mobility model in [1]. In this example, there are four sources (s_1–s_4), three users (u_1–u_3), and an obstacle $o_1 = (2, 2, 4, 3)$. There are overlapping areas between coverage of s_2 and s_3, s_3 and s_4, respectively. The numbers of grid points in these overlapping areas are all exactly 2. Thus they can communicate with each other. In addition, the source s_1 is far away from them, and it has no overlapping area with them; thus, they cannot communicate. However, s_1 will follow a south, east and south path in the following three time units, and at the end of the third time unit, it can communicate with s_4 directly. On the other hand, the user u_3 is within the coverage of s_4, and thus it can communicate with s_2 and s_3 through s_4, while u_2 cannot communicate with any sources and nor with any other users.

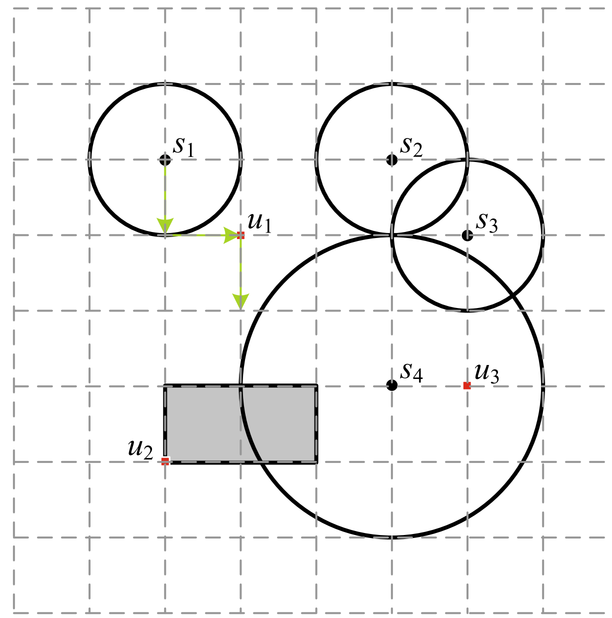

Fig. 1 An example for the mobility model [1]

4 Problems in *P*

4.1 Multiusers simultaneous communication problem

In this section, we investigate a problem called *multiusers simultaneous communication problem*. This problem asks whether k pairs of users can communicate simultaneously throughout the duration of the model without sharing sources. The MUSCP can be formally defined as follows:

Algorithm 1: Preprocessing Algorithm

Input: A mobility model $\mathcal{M}$.
Output: $IA_{i,j}$ $\forall 1 \leq i, j \leq m$ and OR_i $\forall 1 \leq i \leq d$.
begin
for $i = 1$ to m **do**
  compute a new location (x_i^s, y_i^s) at time t for source $s_i \in \mathcal{S}$;
for $i = 1$ to k **do**
  compute new locations (x_i^u, y_i^u) and $(x_i^{u'}, y_i^{u'})$ at time t for uses u_i and u_i', respectively;
for $i = 1$ to m **do**
  compute coverage representation CR_i from $c_i \in \mathcal{C}$ and (x_i^s, y_i^s);
for $i = 1$ to m **do**
  for $j = i + 1$ to m **do**
    $IA_{i,j} = CR_i \cap CR_j$;
for $i = 1$ to d **do**
  compute obstacle representation OR_i from $o_i \in O$;
end.

4.1.1 Multiusers simultaneous communication problem (MUSCP)

Given a mobility model $\mathcal{M} = (\mathcal{S}, \mathcal{D}, \mathcal{U}, \mathcal{L}, \mathcal{R}, \mathcal{V}, \mathcal{C}, \mathcal{O})$, and k pairs of distinct users $\{u_1, u_1'\}, \{u_2, u_2'\}, \ldots, \{u_k, u_k'\}$ from $\mathcal{U}$.

Problem Can all k pairs of users simultaneously communicate throughout the duration of the model without sharing sources?

Algorithm 2: Multiusers Simultaneous Communication Algorithm

Input: A mobility model $\mathcal{M}$, k pairs of distinct users $\{u_1, u'_1\}, \{u_2, u'_2\}, \ldots, \{u_k, u'_k\}$ from $\mathcal{U}$.
Output: YES if all k pairs of users simultaneously communicate throughout the duration of the model without sharing sources. Otherwise, NO.

```
begin
for t = 1 to τ do
    invoke the Preprocessing Algorithm with model M
    for i = 1 to m do
        for j = i + 1 to m do
            if |IA_{i,j}| > 1 then
                n = 1;
                for l = 1 to d do
                    if IA_{i,j} ∩ OR_l = IA_{i,j} then
                        n = 0;
                if n = 1 then
                    E = E ∪ {s_i s_j};
    for i = 1 to m do
        for j = 1 to k do
            if u_j is in the coverage of s_i at time t and the line
            segment between (x_i^s, y_i^s) and (x_j^u, y_j^u) does not
            intersect any obstacle in O then
                E = E ∪ {s_i u_j};
            if u'_j is in the coverage of s_i at time t and the line
            segment between (x_i^s, y_i^s) and (x_j^{u'}, y_j^{u'}) does not
            intersect any obstacle in O then
                E = E ∪ {s_i u'_j};
    for i = 1 to k do
        E = E ∪ {u u_i, v u'_i};
    Invoke the Dinic algorithm [21] to find the maximum number
    κ of node-disjoint paths connecting u and v in the graph
    G(V, E), where V = S ∪ {u_i, u'_i | i = 1, 2, ..., k} ∪ {u, v}.
    if κ < k then
        output NO;
output YES;
end.
```

This problem is closely related to practical application. For example, if there is a limitation of throughput of sources such that one source can only support one connection between users, then different pairs of users who want to communicate simultaneously must find their paths without sharing sources; or there is a robustness requirement of the system, e.g., if one source is damaged, then most connections in the network are required to be uninterrupted. Thus, by allocating connections between different pairs of users into paths without sharing sources, all but one connections will survive when an arbitrary source is damaged. In these scenarios, this problem can be applied in modeling the communication system. By solving the corresponding MUSCP, we can find a strategy to support simultaneous communication of multiple users under limitation of sources or to meet the robustness requirements in wireless ad hoc networks. The theoretical complexity of MUSCP is presented in the following theorem.

Theorem 4.1 *The MUSCP can be solved in $O(\tau m(m+k)^{3/2})$ time on a single-processor machine.*

Proof The key problem in the MUSCP is to find the paths without sharing sources between k pairs of sources. We regard all sources and concerned users as nodes in a graph, and there is an edge between two nodes if and only if they can communicate with each other directly. We now add a new node u connected with all users $u_1, u_2, \ldots, u_k$ and another node v connected with all users $u'_1, u'_2, \ldots, u'_k$. Then, the MUSCP is equivalent to finding k node-disjoint paths connecting u and v in the graph. If there are k such paths at each time unit throughout the duration, then the answer of the MUSCP is YES, and otherwise is NO. More details about solving MUSCP can be seen in Algorithm 2, the multiusers simultaneous communication algorithm. In this algorithm, we invoke the Dinic algorithm [21] as a subroutine to find the maximum node-disjoint paths between u and v in the graph $G(V, E)$ with $|V| = m+2k+2$ and $|E| \leq \binom{m}{2}+2mk+2k$. The Dinic algorithm takes at most $O(|E|\sqrt{|V|})$ time. Considering the main loop, the overall time complexity of the algorithm is $O(\tau m(m+k)^{3/2})$. □

4.2 Longer communication problem

Algorithm 3: Longer Communication Algorithm.

Input: Two mobility models $\mathcal{M}$ and $\mathcal{M}'$.
Output: YES if u_1 and u_2 can communicate for more steps in model $\mathcal{M}$ than they can in model $\mathcal{M}'$. Otherwise, NO.

```
begin
for t = 1 to τ do
    invoke the User Communication Algorithm with model M,
    users u_1, u_2 ∈ U, natural number t;
    if the User Communication Algorithm returns NO then
        output NO;
    else
        invoke the User Communication Algorithm with model
        M', users u_1, u_2 ∈ U, natural number t;
        if the User Communication Algorithm returns NO then
            output YES;
output NO;
end.
```

In the following, we investigate another problem called *longer communication problem*. It asks whether two users can communicate for more steps in one model than in another. The problem models the situation where we want to select a better plan of deploying and scheduling sources which provides users longer communication time. The LCP can be formally defined as follows.

4.2.1 Longer communication problem (LCP)

Given two mobility models $\mathcal{M} = (\mathcal{S}, \mathcal{D}, \{u_1, u_2\}, \mathcal{L}, \mathcal{R}, \mathcal{V}, \mathcal{C}, \mathcal{O})$ and $\mathcal{M}' = (\mathcal{S}', \mathcal{D}, \{u_1, u_2\}, \mathcal{L}, \mathcal{R}', \mathcal{V}', \mathcal{C}', \mathcal{O}')$.

Problem Can u_1 and u_2 communicate for more steps in model $\mathcal{M}$ than they can in model $\mathcal{M}'$?

We define an algorithm called *the longer communication algorithm* to solve LCP. This algorithm has two mobility models and two users as its inputs. It reports YES if the two users can communicate for more steps in the model $\mathcal{M}$ than in $\mathcal{M}'$; otherwise, it reports NO. The main structure of the algorithm is a loop traversing the time from 1 to τ. In each iteration, it uses the user communication algorithm (UCA) proposed in [1] to determine whether the users u_1 and u_2 can communicate in the model $\mathcal{M}$ and $\mathcal{M}'$, respectively. If they cannot communicate in the model $\mathcal{M}$ at the tth iteration, the maximal number of steps they can communicate in the model $\mathcal{M}$ is $t-1$, while in the model $\mathcal{M}'$, it is at least $t-1$. Thus, the algorithm reports NO. If they can communicate in the model $\mathcal{M}$, but cannot communicate in the model $\mathcal{M}'$ at the tth iteration, then the maximal number of steps they communicate in $\mathcal{M}$ must be at least t, while it is $t-1$ in $\mathcal{M}'$, which must be the less one. Thus, the algorithm reports YES. After τ iterations, if the two users can communicate in both $\mathcal{M}$ and $\mathcal{M}'$ in each iteration, then they can communicate for the same number of steps, that is, τ. Then, the algorithm reports NO. Hence the algorithm solves the LCP.

Theorem 4.2 *The LCP can be solved in $O(\max\{d, \tau\}m^4\tau)$ time on a single-processor machine.*

Proof The main structure of the algorithm is a loop consisting of τ iterations. In each iteration, it invokes UCA for two times, of which the time complexity is $O(\max\{d, t\}m^4)$ [1, Theorem 4.1]. Since $t \leq \tau$, the time complexity is less than $O(\max\{d, \tau\}m^4)$. Considering the main loop, the overall time complexity of the algorithm is $O(\max\{d, \tau\}m^4\tau)$. □

5 Problems in NP

In this section, we investigate two problems called *obstacle removal problem* and *user communication, limited number of sources problem*, both of which are finally turned out to be NP-complete.

5.1 Obstacle removal problem

We first study the ORP. It asks whether we can make two users communicate throughout the duration of the model by removing a limited number of obstacles. This problem can be formally defined as follows.

5.1.1 Obstacle removal problem (ORP)

Given a mobility model $\mathcal{M} = (\mathcal{S}, \mathcal{D}, \mathcal{U}, \mathcal{L}, \mathcal{R}, \mathcal{V}, \mathcal{C}, \mathcal{O})$, two designated users u and v from $\mathcal{U}$, and a natural number k.

Problem Can u and v communicate throughout the duration τ of the model if k or fewer obstacles are removed?

In the scenario of military or disaster recovery, it is possible to remove obstacles through transportation or explosion. Then, when we want to find a plan of removing obstacles, through which we can get connected with friendly forces or the victims of disasters, we need to solve the ORP. In order to prove the NP-completeness of the ORP, we first recall a well-known NP-complete problem called the *set covering decision problem* (SCDP) (see [22]) as follows.

5.1.2 Set covering decision problem (SCDP)

Given a set of elements $\{1, 2, \ldots, m\}$ (called *the universe*) and a set $\mathcal{A}$ of n subsets whose union equals the universe, and a natural number k.

Problem Is there a set cover of size k or less whose union equals the universe?

Theorem 5.1 *If $P \neq NP$, then the ORP is NP-complete.*

Proof We use the technique of reduction, and transform an arbitrary SCDP to an ORP in polynomial time.

First consider a SCDP with m elements and the set $\mathcal{A} = \{S_1, S_2, \ldots, S_n\}$, in which S_j is a subset of $\{1, 2, \ldots, m\}$, that is, $S_j \subseteq \{1, 2, \ldots, m\}$ for $j = 1, 2, \ldots, n$. In addition, for each $i \in \{1, 2, \ldots, m\}$, we define $\mathcal{C}_i = \{S \mid i \in S \in \mathcal{A}\} = \{S_{i_1}, S_{i_2}, \ldots, S_{i_h}\}$, where $i_1 < i_2 < \cdots < i_h$ and $h = h_i = |\mathcal{C}_i|$, that is, the number of elements in $\mathcal{C}_i$.

Next, we present the process of transforming this SCDP to an ORP as follows. Firstly select $m+1$ specific sources $s_0, s_1, \ldots, s_m$, and two users u and v, which can only communicate with s_0 and s_m, respectively. Then define n obstacles in the ORP, of which each obstacle $o_j \in \mathcal{O}$ corresponds to a set $S_j \in \mathcal{A}$ in the SCDP for $j = 1, 2, \ldots, n$. The duration of the model $\tau = 1$, that is, we only consider whether the two users can communicate at the first step. In addition, we elaborately construct the ORP such that possible communication can only exist between s_i and s_{i-1}, and whether they can communicate is equivalent to whether the element i is covered in the primary SCDP, for all $i \in \{1, 2, \ldots, m\}$. In the ORP between s_i and s_{i-1} it consists of $(n+3)(|\mathcal{C}_i|+1)-1$ sources and n obstacles, of which an example is shown in Fig. 2. The areas covered with black color are obstacles, and each circle is the coverage area of one source, in which the source is located at the centre. Specifically, there are $|\mathcal{C}_i|$ horizontal paths between s_i and s_{i-1}; each path is composed of $n+1$ sources, of which the coverage areas are overlapped one by one. Thus,

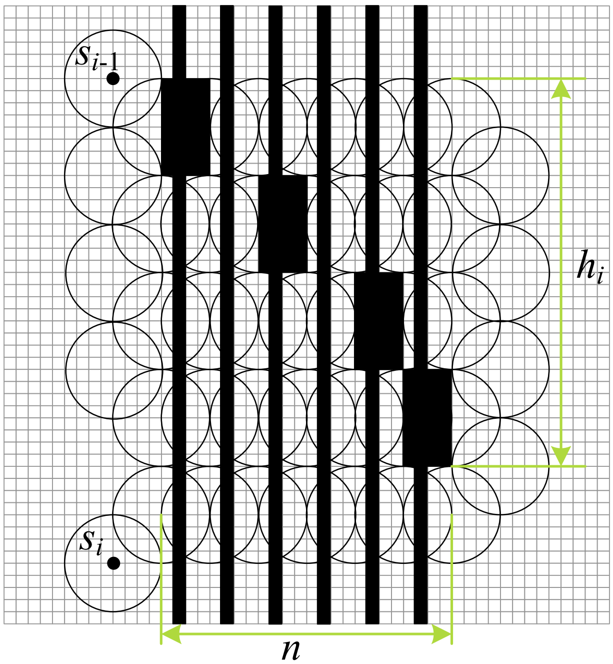

Fig. 2 The part of ORP between s_{i-1} and s_i

there are n overlapping areas in each path. Each overlapping area is also overlapped with one obstacle. For each path, the jth overlapping area from the left is just overlapped with the obstacle $o_j \in \mathcal{O}$ for $j = 1, 2, \ldots, n$. There are two cases for the overlapping between an overlapping area and an obstacle. In the first case, the width of the obstacle is only of one grid wide, and the communication between adjacent sources is not blocked. In the second case, the width of the obstacle is of four grids wide, and it completely covers the overlapping area and thus, the communication between adjacent sources is blocked. We let the obstacle o_{i_l} overlap with the lth path in the second case, that is, blocking the path, for $l = 1, 2, \ldots, |\mathcal{C}_i|$. Meanwhile, we let it overlap with all other paths in the first case, that is, not blocking the path. Thus, the lth path is only blocked by the obstacle o_{i_l}. This describes the construction in ORP between s_i and s_{i-1}. Note that these obstacles exit beyond and go through all parts of the ORP. In this way, we construct the part of ORP between s_i and s_{i-1} for all $i \in \{1, \ldots, m\}$, and obtain the final ORP.

Take Fig. 2 as an example. In this example, we assume $m=10$, $n=6$ and the set $\mathcal{A} = \{\{1, 2\}, \{2, 3, 4\}, \{1, 4, 5\}, \{4, 5, 6\}, \{1, 6, 7, 8\}, \{1, 9, 10\}\}$ in the SCDP. Figure 2 shows the construction of ORP between s_i and s_{i-1} for $i = 1$ with $\mathcal{C}_1 = \{S_1, S_3, S_5, S_6\}$. This construction converts the set cover of $i(=1)$ in the SCDP to the blocking obstacles between s_i and s_{i-1} in the ORP as shown in Fig. 2.

In order to enable communication between s_i and s_{i-1}, one must remove at least one of the obstacles from $o_{i_1}, o_{i_2}, \ldots, o_{i_h}$. In the primary SCDP, in order to cover the element i, one must select at least one set in $\mathcal{C}_i$. Thus, these two problems are equivalent to each other. If there exists in the primary SCDP a set cover of size at most k, i.e., $\{S_{r_1}, S_{r_2}, \ldots, S_{r_p}\}$ where $p \leq k$, then removing their corresponding obstacles $\{o_{r_1}, o_{r_2}, \ldots, o_{r_p}\}$ in the ORP enables users u and v to communicate with each other and vice versa. Hence, the theorem holds. □

Remark. We have modified the mobility model of [1] by merging all adjacent obstacles. However, in the original model of [1], in which each obstacle is a rectangle, the theorem also holds by introducing a little modification to the above proof. In this case, we must maintain the width of the obstacle to be of one grid wide, meanwhile we can elaborately design the coverage areas of sources in each path in such two manners that one can be completely covered by the obstacle but the others cannot.

5.2 User communication, limited number of sources problem

Similarly with MUSCP, in many applications (e.g., disaster recover), resources in the wireless ad hoc network are very limited, such as battery power, throughput of sources, and bandwidth of connections. Thus, in order to provide high-quality and continuous communication service, we should make full use of limited resources. In this section, we focus on such a situation where the number of sources used is limited. Specifically, we refer this problem as the UCLNSP. It requires users to communicate throughout the duration of the model using limited number of sources, of which the formal definition is as follows.

5.2.1 User communication, limited number of sources problem (UCLNSP)

Given a mobility model $\mathcal{M} = (\mathcal{S}, \mathcal{D}, \mathcal{U}, \mathcal{L}, \mathcal{R}, \mathcal{V}, \mathcal{C}, \mathcal{O})$, two designated users u and v from $\mathcal{U}$, a natural number k, and a time bound $t \leq \tau$.

Problem Can users u and v communicate from times 1 to t using at most k sources?

Similarly, this problem is also NP-complete. We also use the technique of reduction from SCDP to show the NP-completeness of the UCLNSP.

Theorem 5.2 *If $P \neq NP$, then the UCLNSP is NP-complete.*

Proof Analogous to the proof of NP-completeness of ORP, we use the technique of reduction, and transform an arbitrary SCDP to an UCLNSP in polynomial time.

Consider a SCDP with m elements and the set $\mathcal{A} = \{S_1, S_2, \ldots, S_n\}$, where S_j is a subset of $\{1, 2, \ldots, m\}$, that is, $S_j \subseteq \{1, 2, \ldots, m\}$ for $j = 1, 2, \ldots, n$. For $i \in \{1, 2, \ldots, m\}$, we define $\mathcal{C}_i = \{S \mid i \in S \in \mathcal{A}\} = \{S_{i_1}, S_{i_2}, \ldots, S_{i_h}\}$, where $i_1 < i_2 < \cdots < i_h$ and $h = |\mathcal{C}_i|$.

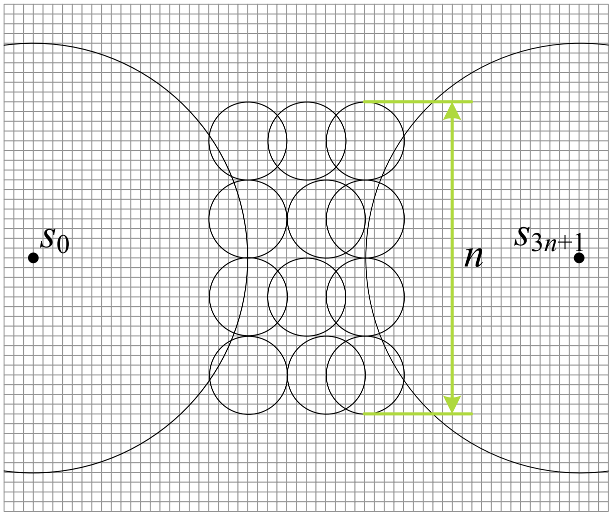

Fig. 3 The UCLNSP at time i

We next show the process of transforming. We construct a UCLNSP with $3n+2$ sources $s_0, s_1, \ldots, s_{3n+1}$ and $\mathcal{O} = \emptyset$. Users u and v are within the coverage areas of s_0 and s_{3n+1}, and they can only communicate with s_0 and s_{3n+1}, respectively; and they do not move in this model. The sources s_0 and s_{3n+1} have huge coverage areas, and there are n paths between them, of which each is composed of three sources and corresponding to a set in $\mathcal{A}$ of the SCDP. In each path, the left-most source can communicate with s_0 and the right-most source can communicate with s_{3n+1}. Each path has two states, obstructed or unobstructed, at each time. As shown in Fig. 3, in the first and third paths, any adjacent sources can communicate with each other, and thus these paths are unobstructed. As for the second and fourth paths, the middle source can only communicate with the right source, leading to the obstruction of the path. Meanwhile, we let the duration of the model τ to be m, which is the number of elements in the universe in the SCDP. For each $i \in \{1, 2, \ldots, m\}$, whether the users can communicate at time i in the UCLNSP is equivalent to whether the element i is covered in the SCDP. We fix the location of s_0 and s_{3n+1} and do not allow them to move, and assign proper directions and speeds to sources $s_1, s_2, \ldots, s_{3n}$ in this model such that the jth path is unobstructed at time i if $i \in S_j$ and it is obstructed otherwise. Note that at time i, the jth path is unobstructed for all $j \in \{i_1, i_2, \ldots, i_h\}$, and the lth path is obstructed for all $l \in \{1, 2, \ldots, n\} \setminus \{i_1, i_2, \ldots, i_h\}$. This completes the construction of UCLNSP.

Take Fig. 3 as an example, which is at time $i(=1)$ of the UCLNSP corresponding to a SCDP with $m = 5$, $n = 4$ and $\mathcal{A} = \{\{1, 2\}, \{2, 3\}, \{1, 4\}, \{4, 5\}\}$. In this example, $\mathcal{C}_1 = \{S_1, S_3\}$. Whether the element 1 is covered in the SCDP is equivalent to whether the users can communicate at time 1 in the UCLNSP, as shown in Fig. 3.

The communication between users u and v at time i must be through one of the $|\mathcal{C}_i|$ unobstructed paths between them. Using such a path means using three sources on this path. In addition, since the users u and v can only communicate with s_0 and s_{3n+1}, respectively, the sources s_0 and s_{3n+1} have to be used in their communication. For such a constructed mobility model with the duration $\tau = m$, we ask a solution of the UCLNSP with at most $3k+2$ sources to be used. This means that we can use at most k paths in total. For the jth path, it is unobstructed at the time corresponding to the elements in S_j. Thus, if there exists a solution to such constructed UCLNSP with $3k+2$ or fewer sources used, then the used paths in the solution induce a set cover with size at most k in the primary SCDP and vice versa. Hence, the theorem holds. □

6 Conclusion and future works

In this paper, based on an existing mobility model for ad hoc networks, we study solutions and complexities of a series of decision problems proposed by Greenlaw et al. [1], which are all derived from real-world applications and will help us to use wireless communications more efficiently in real-world scenario. Specifically, we study four problems, including MUSCP, LCP, ORP and UCLNSP. For MUSCP and LCP, we provide efficient algorithms to solve them and prove that they are P problems, which can be solved in $O(\tau m(m+k)^{3/2})$ and $O(\max\{d, \tau\}m^4\tau)$ time, respectively. On the other hand, for ORP and UCLNSP, by applying reduction from SCDP, we prove that they are both NP-complete, that is, they are intractable, unless $P = NP$.

Only one of the listed problems of Greenlaw et al. [1] still remains open in wireless communication. That is the APPP. We suspect that it is also NP-hard. As for the two problems proved to be NP-complete, it is desired to develop heuristic algorithms to solve them in reasonable time. At the same time, we will focus on modifying the mobility model to get a more realistic one, such as introducing the SINR model into communication between sources and users, or extending to three-dimensional case. Thus, we can extend our studied problems into a more general situation.

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