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A survey of physics-informed AI for complex urban systems

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ABSTRACT

Urban systems are typical examples of complex systems, where the integration of physics-based modeling with artificial intelligence (AI) presents a promising paradigm for enhancing predictive accuracy, interpretability, and decision-making. In this context, AI excels at capturing complex, nonlinear relationships, while physics-based models ensure consistency with real-world laws and provide interpretable insights. We provide a comprehensive review of physics-informed AI methods in urban applications. The proposed taxonomy categorizes existing approaches into three paradigms-Physics-Integrated AI, Physics-AI Hybrid Ensemble, and AI-Integrated Physics-and further details seven representative methods. This classification clarifies the varying degrees and directions of physics-AI integration, guiding the selection and development of appropriate methods based on application needs and data availability. We systematically examine their applications across eight key urban domains: energy, environment, economy, transportation, information, public services, emergency management, and the urban system as a whole. Our analysis highlights how these methodologies leverage physical laws and data-driven models to address urban challenges, enhancing system reliability, efficiency, and adaptability. By synthesizing existing methodologies and their urban applications, we identify critical gaps and outline future research directions, paving the way toward next-generation intelligent urban system modeling.

1. Introduction

The integration of physics and AI is driving a paradigm shift in scientific research. The awarding of the 2024 Nobel Prizes in Physics [1] and Chemistry [2] to AI-related achievements and scientists highlights this trend. This not only reflects the far-reaching impact of AI methodologies but also underscores the significant potential of combining physics with data science in addressing complex system problems [3]. Physics provides a foundational framework for explaining the world through mathematical rigor and theoretical elegance, while AI has become a vital tool in modern science due to its ability to capture data patterns and support predictive modeling. These two paradigms excel in different domains. For example, Newtonian mechanics formulates natural laws through precise mathematical expressions [4], whereas fields such as protein structure prediction [2] and weather forecasting [5] rely heavily on efficient data-driven modeling [6].

Urban systems involve multidimensional complexity and dynamic interactions, posing challenges that are difficult to address through traditional physics-based or purely data-driven approaches alone [7,8]. In particular, for tasks involving nonlinear relationships, such as traffic flow or population migration, AI methods have demonstrated strong

capabilities in extracting complex patterns from large-scale historical data and achieving accurate predictions. AI is also effective in handling large-scale, heterogeneous datasets, and is widely applied in domains such as intelligent transportation, power grids, and emergency response, where it supports real-time decision-making and enhances system responsiveness [9].

Many urban problems are fundamentally governed by physical mechanisms, such as fluid dynamics [10], heat transfer [11], pollutant dispersion [12], gravity-driven flows [13], and social force dynamics [14]. In such contexts, physics-based models play a critical role in ensuring the reliability and consistency of model outputs, offering particular advantages in multi-scale, dynamic systems and in data-scarce scenarios where prior knowledge mitigates the need for large datasets. To address the complex challenges of urban systems, integrating physics with AI provides a complementary solution: physics offers a rigorous theoretical foundation, while AI leverages data to uncover latent patterns and compensate for the limitations of purely physics-based models [15,16]. For example, in intelligent transportation systems, AI can optimize traffic signal control while physics-based models maintain the physical consistency of traffic flow; in pollution forecasting, combining meteorological models with AI techniques enhances predictive accuracy. Such

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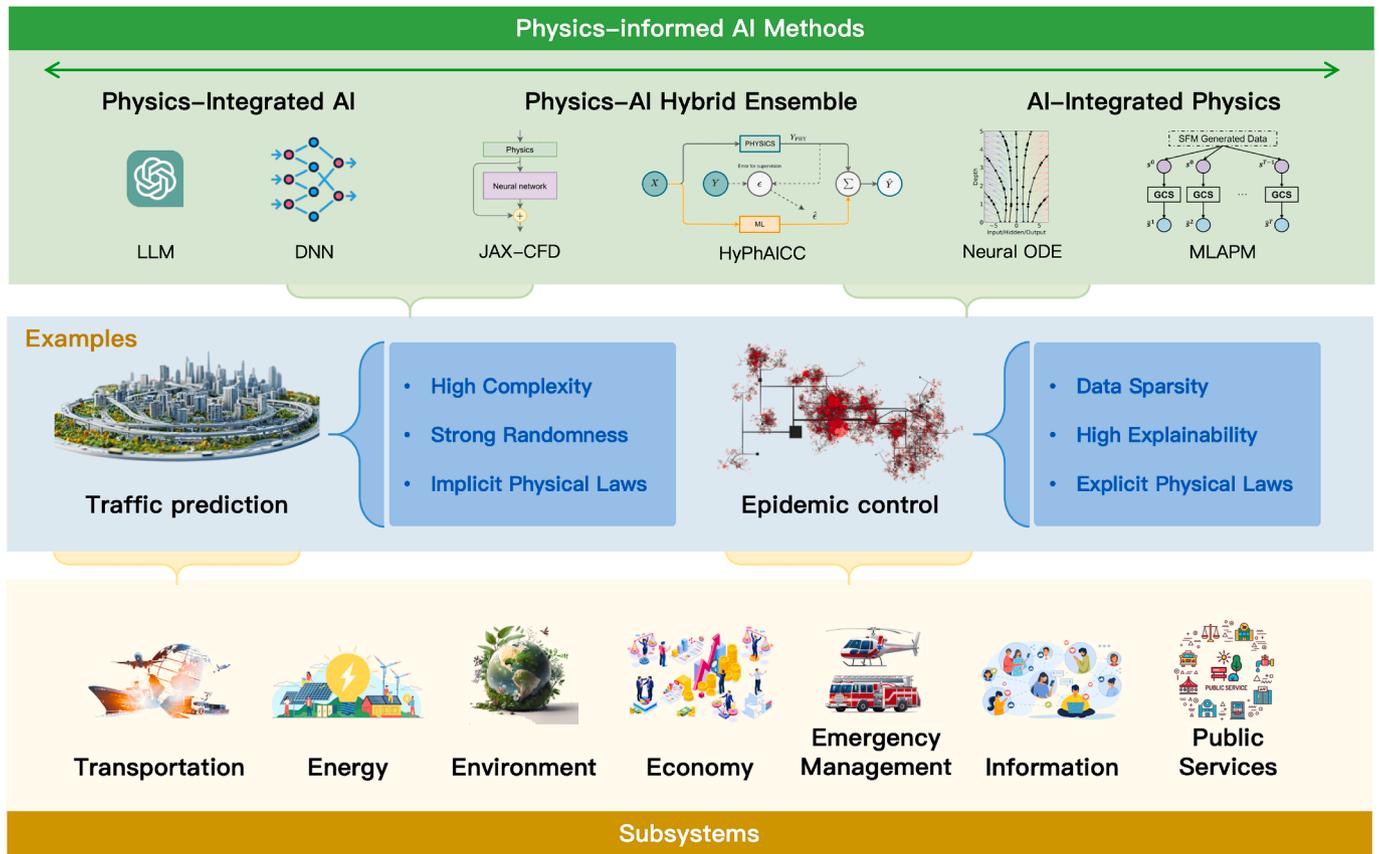


Fig. 1. Classification of PIAI methods and their applications in urban systems. The taxonomy categorizes methods into three paradigms-Physics-Integrated AI, Physics-AI Hybrid Ensemble, and AI-Integrated Physics-and reviews their applications across urban subsystems.

physics-AI fusion supports more comprehensive and reliable decision-making, with applications spanning climate adaptation, resource management, and beyond.

The integration of physics and AI varies across urban tasks. Traffic prediction, driven by abundant sensor data and the stochasticity of human behavior and multimodal transportation, typically favors data-driven approaches. Conversely, epidemic control relies more heavily on explicit physical modeling of disease dynamics (e.g., SIS, SIR models [17]). As illustrated in Fig. 1, we categorize physics-informed AI methods into three paradigms-Physics-Integrated AI, Physics-AI Hybrid Ensemble, and AI-Integrated Physics-reflecting distinct integration strategies.

Most existing reviews focus on technical classifications of physics-informed AI methods, typically from an AI-centric perspective, emphasizing algorithmic differences. However, these studies generally lack a systematic classification based on the direction and degree of integration between physics and AI, and rarely provide a focused review on urban systems. As shown in Table 1, this review introduces a new taxonomy-categorizing methods into three paradigms based on their integration strategy-and applies this framework to systematically examine the development of physics-informed AI methods in urban systems.

Furthermore, we divide urban systems into seven core subsystems, ranging from domains of energy and environment to upper-layer services, including public service and emergency management. We systematically examine the interplay between physical mechanisms and data characteristics within each subsystem and explore corresponding modeling requirements for fusion. The structure of this review is as follows: Chapter 2 introduces the overarching framework for physics-informed AI methods and details seven representative fusion methods; Chapter 3 presents the seven key urban subsystems and their underlying physical principles, highlighting the necessity of incorporating physical

knowledge into complex urban modeling; Chapter 4 offers a subsystem-by-subsystem review, discussing major research challenges and summarizing practical implementations of existing fusion methods; Chapter 5 concludes the review with a discussion and outlook on future research directions; Chapter 6 concludes the review.

2. Physics-informed AI methods

The integration of AI with physics-based modeling has emerged as a pivotal research direction for tackling complex system challenges. Traditional physics-based models, grounded in explicit mathematical formulations and fundamental laws, provide physically consistent predictions but often face computational inefficiencies and challenges in handling multi-scale dynamics. In contrast, AI-driven methods excel at capturing complex, nonlinear patterns from large-scale data but lack inherent physical constraints, limiting their reliability in data-scarce or extrapolation scenarios.

To bridge these complementary strengths, physics-informed AI (PIAI) methods integrate physics-based knowledge with AI techniques to enhance predictive accuracy, generalization, and computational efficiency. We categorize PIAI methods into three paradigms-(1) Physics-Integrated AI, (2) Physics-AI Hybrid Ensemble, and (3) AI-Integrated Physics-each representing distinct integration strategies. Specifically, Physics-Integrated AI includes Physics-Informed Neural Networks (PINN) with Loss Function, Weight Initialization, and Architecture Design; Physics-AI Hybrid Ensemble comprises Sequential Physics-AI Ensemble and Parallel Physics-AI Ensemble; and AI-Integrated Physics covers AI-Discovered Physics Model and Neural Physics Model. As illustrated in Fig. 2, these methodological approaches enable the synergistic integration of physics and AI to advance predictive modeling and decision support in complex urban systems. Detailed

Table 1

Comparison with existing surveys. This paper provides a comprehensive review of PIAI methods and their applications in urban systems.

Survey	Venue and Year	Main Focus	Deficiency
[18]	RMP, 2019	Physical problems	Limited to physics-based analysis
[19]	JSC, 2022	Technical methods	Not targeted at urban systems
[20]	Arxiv, 2022	Technical methods	Not targeted at urban systems
[21]	Nat. Rev. Phys., 2021	Physical problems	Not targeted at urban systems
[22]	IEEE TPWRS, 2020	Power flow	Limited scope
[23]	RESS, 2023	Reliability and system safety	Limited scope
[24]	ESWA, 2024	Anomaly and condition monitoring	Limited scope

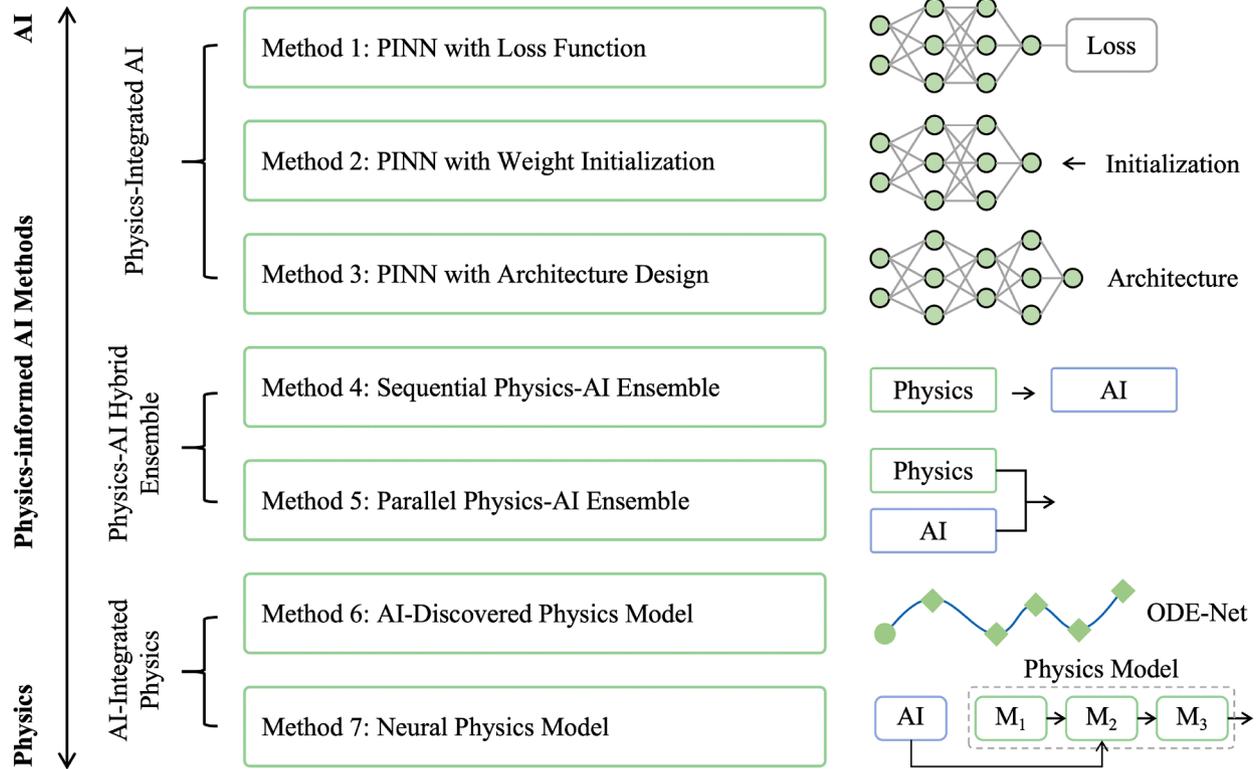


Fig. 2. Physics-informed AI methods are categorized into seven types based on the degree of reliance on AI and physics.

mechanisms and representative case studies for each method are provided in Appendix A, where the correspondence between figures and the main manuscript is also clarified. A structured comparison with conventional data-driven AI approaches—including differences in scope, modeling assumptions, and practical implications—is presented in Appendix B. Furthermore, Table 2 provides a concise overview of the seven representative PIAI methods, summarizing their core principles, advantages, limitations, and application domains for practical reference, while detailed explanations and extended analyses are available in Appendix G.

2.1. Physics-integrated AI

This category represents AI models that incorporate physical constraints during training or within network architecture. Methods embed governing equations, conservation laws, or symmetry properties into the learning pipeline, thereby promoting physical consistency.

2.1.1. Method 1: PINN with loss function

PINN with loss function design integrates physical constraints into deep learning models by encoding fundamental laws as additional loss

terms, as illustrated in Fig. 3a [25]. This helps ensure physically consistent predictions throughout optimization, improving model stability, generalization, and extrapolation, especially in cases with limited or noisy data.

As early as the 1990s, researchers began exploring the use of simple neural networks to approximate solutions of PDEs, with pioneering work by Dissanayake and Lagaris introducing the integration of neural networks with boundary conditions [26]. In the 2000s, advances in computational power and tools such as automatic differentiation enabled the application of deeper and more complex neural networks to PDE problems [27]. In 2017, Raissi et al. [28] systematically proposed the PINN framework, embedding PDE residuals into the neural network loss function, thereby enabling the simultaneous treatment of forward and inverse problems in an unsupervised setting—marking the formal emergence of modern PINNs. Since then, a variety of variants have rapidly emerged, including conservative PINNs (CPINNs) [29], variational hp-PINNs [30], and physics-constrained neural networks (PCNNs) [31], designed to address diverse modeling requirements in physical systems. In recent years, theoretical studies on the generalization, convergence, and error analysis of PINNs have gradually progressed, laying a preliminary

Table 2
Seven representative PIAI methods: Principle, strengths, limitations, and typical urban applications.

Method	Principle	Strengths	Limitations	Applications
M1	Add physical laws to the loss	Physically consistent under sparse data	Weight-sensitive; weak inference constraint	Power flow, air quality, heat transfer
M2	Use physical simulations for parameter initialization	Fast convergence; less data required	Relies on simulation accuracy; possible bias	Energy, storage control, traffic signals
M3	Embed physics into model architecture	Intrinsic consistency; interpretable and transferable	Needs customized design; limited flexibility	Energy systems, traffic flow, communication
M4	Sequentially couple physics and AI model	High accuracy with reduced computation	Complex coupling; non-differentiable solvers	CFD, congestion, hazard simulation
M5	Parallel physics-AI modeling of processes	Robust and generalizable for open systems	Difficult alignment; high computational cost	Air dispersion, flooding
M6	Discover governing equations from data	Derives explicit, interpretable laws	Sensitive to noise; library dependence	Economics, public services, social dynamics
M7	Embed neural operators into physics model	Strong physical fidelity; stable and deployable	Complex design; costly PDE solving	Traffic, crowd motion, diffusion

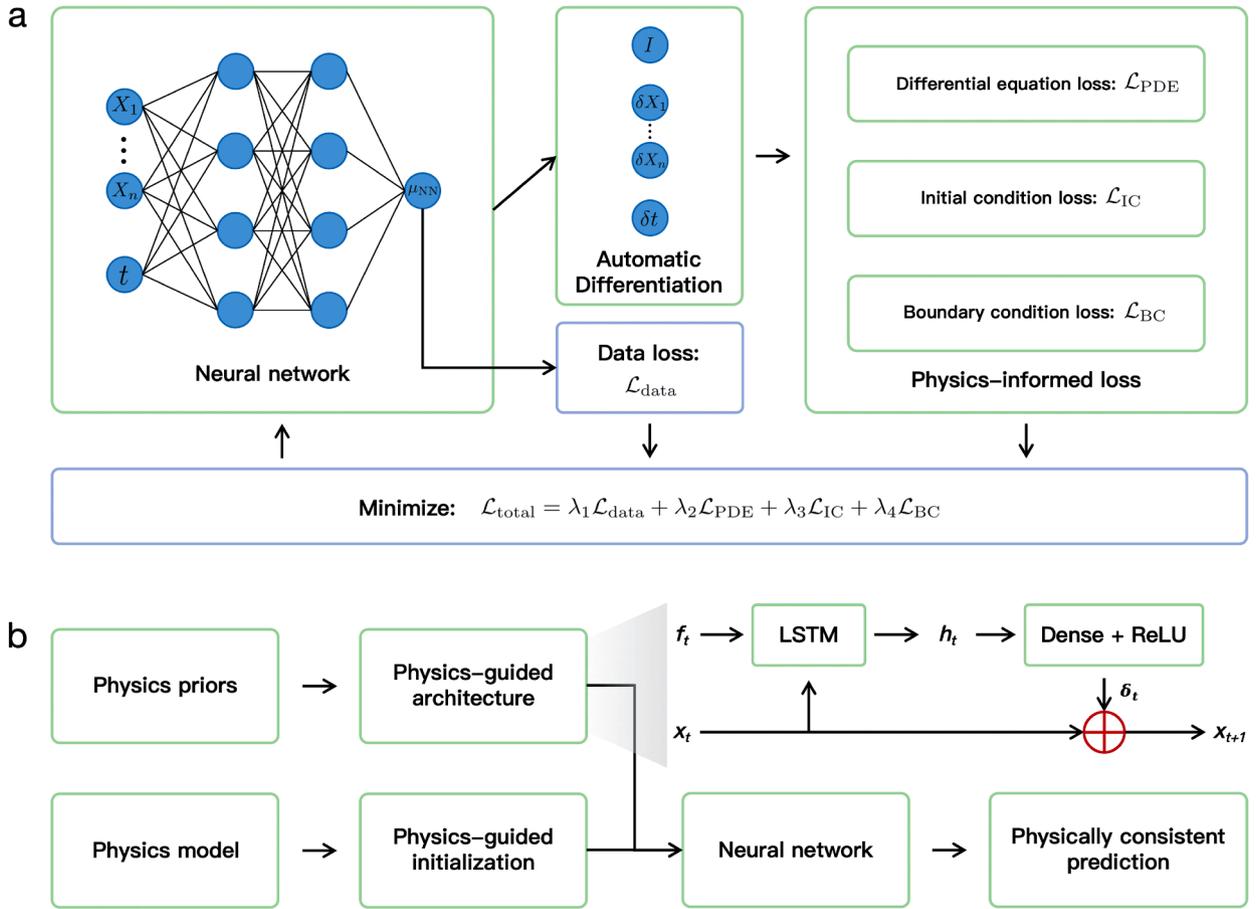


Fig. 3. Illustration of two Physics-Integrated AI methods: (a) PINN with weight initialization and (b) PINN with architecture design. In the lower pathway, the PINN with weight initialization leverages outputs from a physics-based model to initialize neural parameters, providing a physically meaningful starting point that guides convergence toward feasible solutions. In the upper pathway, the PINN with architecture design embeds physical priors directly into the network structure; an LSTM-based model predicts the incremental update δ_t of the system state through a non-negative mapping, ensuring monotonic and physically consistent evolution. Together, these methods incorporate physical knowledge at both the parameter and structural levels, yielding models that are more stable, interpretable, and aligned with underlying physical dynamics.

mathematical foundation for their role as a tool in scientific machine learning.

2.1.2. Method 2: PINN with weight initialization

PINN with weight initialization leverages prior physical knowledge to optimize the initial state of neural networks, thereby improving training efficiency, stability, and generalization, as illustrated in Fig. 3b [32]. By embedding physical consistency into the initialization process,

models can exhibit better convergence properties and reduce reliance on extensive real-world data.

Early studies demonstrated that pretraining with synthetic data generated by physics-based models can significantly enhance model robustness and generalization to unseen scenarios [33]. This approach has since evolved to include techniques such as transfer learning, self-supervised pretraining [32], and physics-informed Gaussian processes [34]. These methods enable neural networks to incorporate domain-

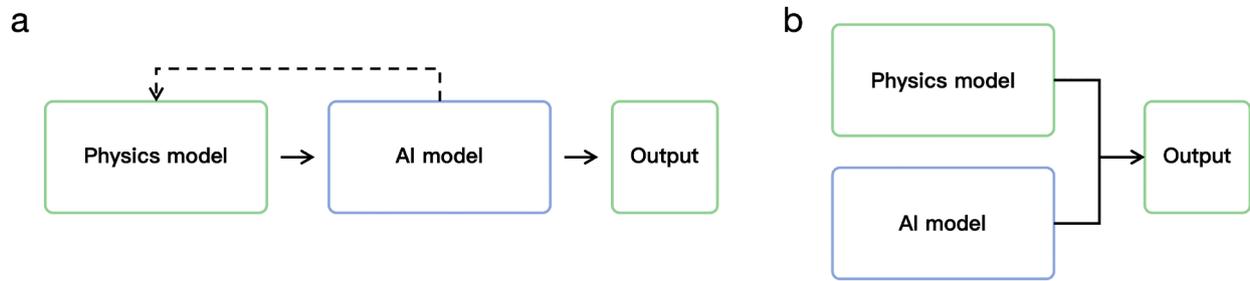


Fig. 4. The figure illustrates two integration strategies: (a) pipeline integration, where models are connected in sequence; (b) prediction fusion, where models run in parallel and their outputs are combined.

specific physical knowledge from the outset of training, accelerating convergence and improving predictive accuracy in data-scarce environments.

2.1.3. Method 3: PINN with architecture design

PINN with architecture design incorporates physical laws and domain knowledge directly into model structures, enhancing both interpretability and physical fidelity, as illustrated in Fig. 3b [35]. By embedding physical constraints into the network architecture, models can better represent dynamical systems and achieve physically consistent predictions.

Initial efforts in this direction focused on embedding intermediate physical variables or fixing structural parameters [35]. Subsequent advances have led to architectures that encode physical symmetries [36], Hamiltonian structures [37], PDE-based mappings [38], and Fourier operators [39]. These designs constrain the solution space to physically plausible regimes, resulting in more efficient training, stronger generalization, and improved interpretability [37]. As a result, physics-guided architecture design has become a key paradigm in building trustworthy models for scientific machine learning and complex system modeling.

2.2. Physics-AI hybrid ensemble

These approaches couple physics-based and AI-based models into cooperative ensembles. Sequential coupling refers to models connected in a pipeline with a configurable order between the physics and AI components; parallel coupling denotes co-evolution and fusion of the two branches.

2.2.1. Method 4: Sequential physics-AI ensemble

Sequential physics-AI ensemble integrates the strengths of both approaches through staged or alternating processes, ensuring physical consistency while leveraging the flexibility of data-driven learning, as illustrated in Fig. 4a [40]. This method is particularly effective for complex dynamical systems and computationally intensive simulations, where physics-based models provide structured representations, and AI compensates for unmodeled nonlinearities.

The sequential hybridization of AI and physics-based models has evolved progressively from error correction to predictive enhancement. The earliest form, known as residual modeling, employs machine learning to predict the systematic bias of physics-based models, which is then added as a correction term to the original output [41]. This was followed by more general pipeline architectures, in which the output of a physics-based model is used as input to a machine learning model, enabling data-driven components to further improve predictive accuracy and generalization. In certain temporal modeling tasks, this approach has been extended to iterative structures at each time step, where the physics-based model provides fundamental constraints and the AI model refines predictions accordingly. The two models operate alternately over time, jointly guiding the system's evolution, allowing the hybrid model to maintain physical consistency while adapting to complex patterns in the data [42].

2.2.2. Method 5: Parallel physics-AI ensemble

Parallel physics-AI ensemble is a modeling strategy in which both components independently process the same inputs and their outputs are subsequently combined through fusion mechanisms to generate final predictions, as illustrated in Fig. 4b. Compared to sequential coupling, this structure is more symmetric and flexible, allowing the integration of physical constraints with data-driven corrections, and is widely used in complex systems where data are limited or physics models are incomplete [43].

Initially employed to capture complementary behaviors across physical scales, this approach has evolved to incorporate dynamic weighting schemes based on spatial, temporal, or error-based criteria [44]. In tasks such as PDE solving, hybrid architectures have emerged where neural networks and analytical solvers jointly and independently solve different components of the system, improving both accuracy and stability [45]. The core principle of parallel hybridization lies in using physics-based models to provide structural and theoretical constraints, while AI models compensate for residual errors and capture fine-grained dynamics. The two components reinforce each other within a unified framework. Representative applications include air quality forecasting that fuses pollutant transport models with deep learning corrections [46], biochemical reaction systems where AI refines physical residuals [47], and fatigue crack growth modeling where AI adjusts for environmental effects to improve structural health predictions [48].

2.3. AI-integrated physics

In this complementary paradigm, AI directly participates in the physical modeling process by discovering governing equations or completing unknown mechanisms from data.

2.3.1. Method 6: AI-Discovered physics model

AI-Discovered Physics Models aim to automatically identify the governing mathematical structures of dynamical systems directly from observational data, or to approximate these structures when the underlying equations are only partially known, as illustrated in Fig. 5a. This direction has advanced rapidly in recent years, marking a shift from traditional handcrafted modeling toward interpretable, data-driven scientific discovery.

Early efforts relied on heuristic strategies and expert knowledge to rediscover known empirical laws from synthetic data. For instance, the BACON system used enumeration and pattern matching to recover algebraic relations without differential forms [49]. These approaches were later extended to real-world dynamical systems such as ecological modeling, where differential equations are inferred directly from time-series data [50]. Subsequent developments introduced sparse regression and symbolic modeling as core techniques. The SINDy framework (Sparse Identification of Nonlinear Dynamical Systems) uses sparse selection from a predefined dictionary of nonlinear functions, coupled with numerical derivative estimation, to construct governing differential equations automatically [51]. More recently, hybrid methods have combined neural networks with symbolic regression. For example, Udrescu et al.

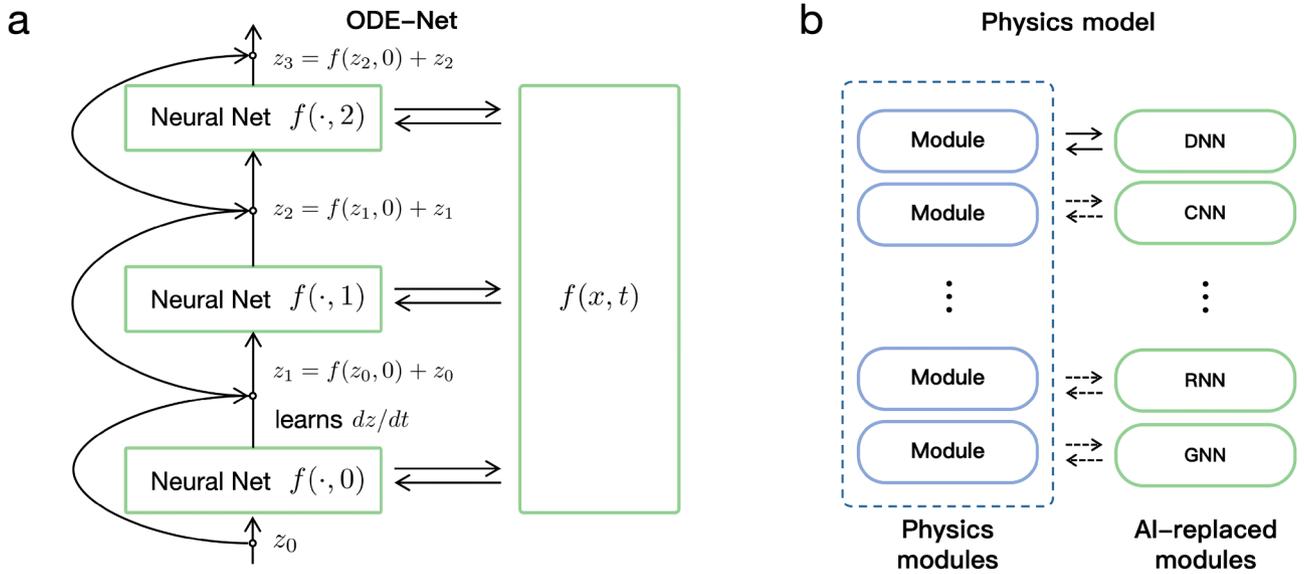


Fig. 5. Illustration of two AI-Integrated Physics methods. (a) AI-Discovered Physics Model: a neural module $f_{\theta}(\cdot, t)$ learns the time derivative of the system state from observational data within an ODE-Net, approximating the unknown dynamics $f(x, t)$. The sequential computation $z_{t+1} = z_t + f_{\theta}(z_t, t)$ represents continuous temporal evolution and supports data-driven discovery of governing equations. (b) Neural Physics Model: the explicit physical equation is retained while poorly characterized components (e.g., closures or interaction terms) are replaced by learnable neural operators. Training couples data fit with physics constraints (e.g., conservation and boundary conditions), yielding adaptive yet interpretable dynamics.

proposed a recursively structured model that first identifies symmetry and separability in the data, then constrains the symbolic search space, improving both discovery efficiency and physical interpretability [52]. Overall, this methodology represents a shift from black-box prediction to white-box modeling, offering strong interpretability, generalizability, and theoretical insight—especially in high-dimensional dynamics, multi-source data integration, and scientific hypothesis generation.

2.3.2. Method 7: Neural physics model

Neural Physics Models retain the overall structure of physics models while replacing specific physics modules—typically those that are difficult to model, inaccurate, or computationally expensive—with AI-replaced modules. By embedding data-driven components within a physically consistent framework, they enhance nonlinear expressiveness and generalization while preserving interpretability and structural reliability, as illustrated in Fig. 5b.

Early applications emerged in computational physics and engineering. Parish et al. replaced turbulence closure terms in RANS models with neural networks to reduce systematic bias in complex flows [53], while Hamilton et al. and Zhang et al. used ML to substitute sub-equations or optimization steps in power system estimation, improving accuracy and real-time performance [54]. More recently, this approach has been extended to social and dynamical systems. Examples include replacing interaction terms in pedestrian dynamics models, using neural networks to learn Koopman eigenfunctions without manual observable selection, and employing physics-informed GANs to substitute subgrid closure models in LES, improving efficiency while ensuring physical consistency [55].

3. Complex urban systems and involved physical laws

Urban systems are complex and dynamic, consisting of various sub-systems that interact with each other to maintain the function and development of a city. Based on existing studies [56], in this survey, we broadly category the research areas in urban systems into energy, environment, economy, transportation, information, public service, and emergency management domains, as shown in Fig. 6. Many elements in these domains are governed by physical laws, as shown in Table 3. Each

physical model represents a crucial aspect of urban dynamics, ranging from power flow in electricity grids to pollution dispersion in the atmosphere, consumer behavior in economic markets, and traffic flow on transportation networks. The study of these physical laws provides insights into the behavior and optimization of urban systems, helping to design more efficient, sustainable, and resilient cities. In this section, we present an overview of each domain and several typical physical models therein. Beyond theoretical modeling, we also analyze the practical benefits of PIAI in real urban systems. A concise summary of its measurable improvements across urban subsystems is provided. Detailed mechanisms and performance analyses are presented in Appendix C.

3.1. Energy

The energy system is fundamental to urban operation, encompassing electricity and gas networks that supply power to residential, commercial, and industrial sectors. These systems are governed by physical laws that capture how energy flows, transforms, and balances within a city. For electricity networks, power flow equations describe the relationship between voltage angles, power injections, and network admittance [57], reflecting conservation laws and the dynamics of electrical interactions. Similarly, gas networks are modeled through pressure balance equations [58], which embody fluid dynamics principles to ensure stable distribution across the system. These physical laws—covering energy conservation, resistance, and flow dynamics—form the foundation for managing and optimizing urban energy infrastructure, highlighting the interconnected and law-governed nature of energy systems in cities. PIAI methods are essential in optimizing power generation, transmission, and consumption, particularly with the integration of renewable energy sources, by enhancing system efficiency and stability [70].

3.2. Environment

Urban environmental systems involve complex physical processes governing water flow, air pollution, and noise propagation. These phenomena are described by foundational physical laws that help us understand and manage environmental quality. For instance, water systems follow conservation laws as captured by the Storm Water Management Model (SWMM) [59], which relates rainfall, inflow, and outflow

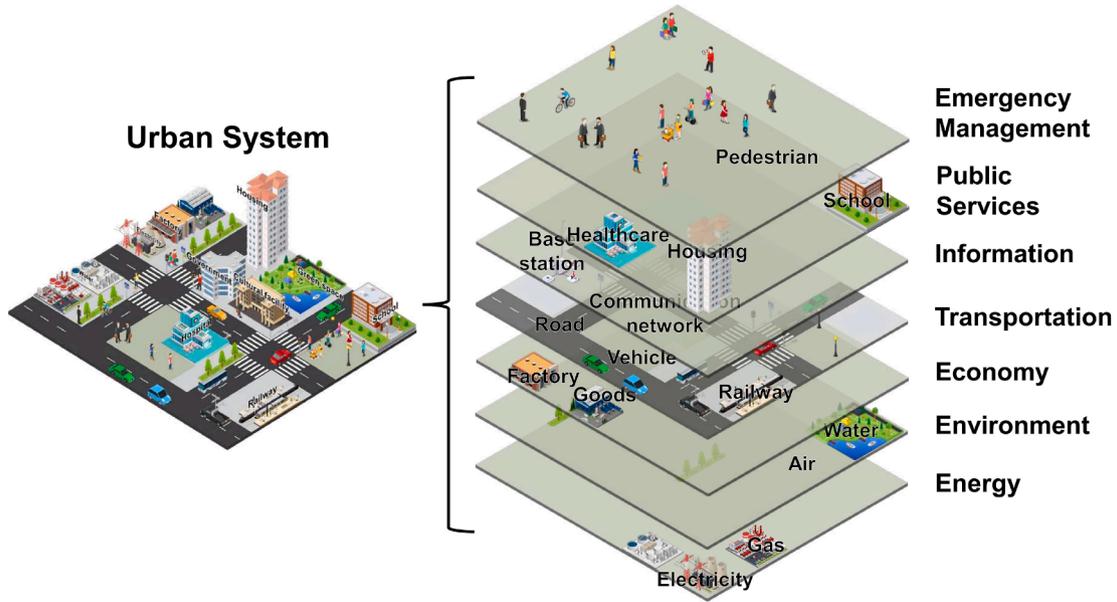


Fig. 6. Sub-system of urban systems.

Table 3
Typical physical laws in different urban systems.

Domain	Element	Physical model	Equation
Energy	Electricity	Power Flow Calculation [57]	$\frac{d^2\theta_i}{dt^2} + \gamma \frac{d\theta_i}{dt} = P_i - \kappa \sum_{j=1}^n A_{ij} \sin(\theta_i - \theta_j)$
	Gas	Gas Network Pressure Balancing Model [58]	$\sum_{j \in \mathcal{N}^{-(i)}} C_{ij} \cdot \text{sgn}(p_i - p_j) \cdot \sqrt{ p_i^2 - p_j^2 } = d_i$
Environment	Water	Storm Water Management Model [59]	$\frac{\partial h}{\partial t} = \sum_i Q_{ri} + \sum_i Q_{k_i} - Q_o$
	Air	Gaussian plume model [60]	$C(x, y, z) = \frac{Q}{2\pi\sigma_y\sigma_z u} \exp\left(-\frac{(y-y_0)^2}{2\sigma_y^2}\right) \exp\left(-\frac{(z-z_0)^2}{2\sigma_z^2}\right)$
Economy	Goods	Network Flow Model [62]	$\sum_{j \in \mathcal{N}^{-(i)}} f_{ji} - \sum_{k \in \mathcal{N}^{+(i)}} f_{ik} = b_i$
	Consumer	Huff model [63]	$P_{ij} = \frac{A_i \cdot d_{ij}^{-\beta}}{\sum_{k \in \mathcal{N}} A_k \cdot d_{ik}^{-\beta}}$
Transportation	Railway	OpenTrack [64]	$F_{\text{resistive}} = C_r \cdot m \cdot g + C_d \cdot \frac{1}{2} \cdot \rho \cdot A \cdot v^2$
	Road	SUMO [65]	$\frac{dv}{dt} = a(v, \rho) = a_{\text{max}} \left(1 - \left(\frac{v}{v_{\text{max}}}\right)^\beta\right)$
	Vehicle	Car Following and Lane Change Model [66]	$\Delta x = \frac{1}{\rho} - \frac{v}{\beta} - \frac{\gamma \cdot \rho \cdot (v - v_{\text{leader}}(t))}{\beta}$
Information	Communication Network	Communication Network Model [67]	$C = B \log_2 \left(1 + \frac{S}{N}\right)$
	Base Station	Ray Tracing Model [68]	$L(d) = L_0 + 10n \log_{10} \left(\frac{d}{d_0}\right) + X_\sigma$
Public Services	Public health	SIR Model [69]	$\frac{dS}{dt} = -\beta \frac{SI}{N}$
			$\frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I$
Emergency Management	Pedestrian	Social Force Model [14]	$\frac{dR}{dt} = \gamma I$
			$m \cdot \frac{d^2 \mathbf{r}_i}{dt^2} = -k_{\text{goal}}(\mathbf{r}_i - \mathbf{r}_{\text{goal}}) + \sum_{j \neq i} \frac{\gamma}{ \mathbf{r}_i - \mathbf{r}_j ^2} \hat{\mathbf{r}}_{ij} + \sum_{k \in \text{obstacles}} \frac{\beta}{ \mathbf{r}_i - \mathbf{r}_k ^2} \hat{\mathbf{r}}_{ik}$

to changes in water depth across drainage areas. Air quality is governed by diffusion and advection principles, modeled by the Gaussian plume equation [60], which predicts pollutant dispersion based on emission rates, wind speed, and atmospheric conditions. Noise in urban areas follows the physics of wave propagation, with sound levels decaying logarithmically over distance [61]. These models illustrate how environmental subsystems are shaped by universal physical laws, providing a scientific basis for pollution control and sustainable urban planning. In summary, the integration of PIAI methods in urban environmental systems is critical for optimizing air quality, water resource management, soil dynamics, and waste management. These approaches combine physical laws with AI techniques to enhance urban environmental

governance and sustainability, providing actionable insights for improving urban ecological systems' resilience.

3.3. Economy

Urban economies involve complex interactions between supply, demand, and consumer choice, many of which can be described using physical analogies and laws. The distribution of goods and services is often modeled through network flow frameworks [62], reflecting principles of flow conservation across supply chains and trade routes. These models capture how resources move through urban space under constraints similar to those in physical systems. Meanwhile, consumer

behavior-central to economic dynamics-is often represented by probabilistic choice models like the Huff model [63], where decisions are influenced by factors such as distance and attractiveness, resembling potential fields and decay functions in physics. These physical perspectives offer a unifying lens to understand economic behavior in cities, enabling more systematic planning and optimization of urban markets. PIAI in urban economics addresses key challenges such as optimizing resource allocation, forecasting demand, and improving financial systems. These approaches enhance system intelligence by combining physical laws with AI techniques, leading to more efficient economic decision-making processes [71].

3.4. Transportation

Transportation systems are essential to urban functionality, enabling the movement of people and goods through roadways, railways, and multimodal networks [72]. Underpinning these systems are physical laws that describe motion, resistance, and flow dynamics. For railways, mechanical forces such as rolling resistance and aerodynamic drag govern train dynamics and energy consumption [64]. In road networks, traffic flow is shaped by vehicle interactions and density-dependent dynamics, often modeled through equations that relate speed, acceleration, and spacing [65]. These models, grounded in classical mechanics and fluid analogies, reflect how individual behaviors and collective flows emerge from fundamental physical interactions. By leveraging such physical laws, urban planners and engineers can simulate, analyze, and optimize transportation networks for improved efficiency, safety, and resilience. PIAI in urban transportation addresses key challenges such as traffic flow prediction, state estimation, and real-time control. By combining physical models of traffic dynamics with AI techniques, these methods enhance the efficiency, safety, and adaptability of transportation systems [73].

3.5. Information

The Information domain underpins the digital infrastructure of urban systems, enabling real-time communication and coordination across sectors. Governed by fundamental laws of information theory and signal propagation, this domain ensures the efficient transfer of data essential for urban operations. Models such as the Shannon capacity formula [67] capture the theoretical limits of data transmission based on bandwidth and signal-to-noise ratio, while ray tracing and path loss models [68] describe how signals attenuate over space and interact with the urban environment. These physical principles reveal how information flows are constrained and optimized, forming the basis for reliable, scalable communication networks in smart cities. PIAI in urban information systems focuses on improving the prediction of information diffusion and optimizing the modeling of opinion evolution. By combining physical laws with machine learning techniques, these approaches enhance the accuracy of social network content predictions and improve decision-making in dynamic environments [74].

3.6. Public services

Urban public services-including health, safety, and social welfare-are deeply influenced by collective human dynamics, many of which can be captured through physical models. A prominent example is the SIR model [69], which uses differential equations to describe the spread of infectious diseases based on interactions among susceptible, infected, and recovered individuals. This model reflects underlying principles of dynamic systems and population flows, allowing us to quantify how contagion propagates and stabilizes over time. Such physical representations are key to planning effective responses in health crises, allocating resources, and ensuring resilience in the face of emergencies. PIAI methods play a crucial role in enhancing urban public services, particularly in

public health, safety, and infrastructure planning. These approaches integrate physical models with AI to improve epidemic forecasting, predict crowd dynamics, and optimize urban service delivery, thus contributing to the overall resilience and efficiency of urban systems [55].

3.7. Emergency management

Emergency management in urban systems addresses the response to and mitigation of disasters, where the movement and interaction of individuals are governed by physical principles of force and motion. A widely used model in this domain is the Social Force Model [14], which treats pedestrian behavior as driven by virtual forces-toward goals, away from other individuals, and around obstacles-analogous to Newtonian dynamics. These forces encapsulate concepts such as repulsion, attraction, and inertia, allowing for the simulation of crowd behavior under stress. By grounding human movement in physical analogies, such models provide a quantitative framework for optimizing evacuation strategies and improving safety in complex urban environments. PIAI in urban emergency management focuses on improving the prediction and management of human mobility and disaster dynamics. By integrating physical models with machine learning techniques, these approaches enhance the accuracy of evacuation planning, resource distribution, and disaster mitigation strategies [75].

4. Physics-informed AI in complex urban systems

4.1. Energy

4.1.1. Research focus and challenges in power system

Research focus. As renewable energy integration increases, research has focused on improving power system efficiency and reliability. As illustrated in Fig. 7, we focus on three stages in power system, namely power generation, power transmission, and power consumption. In power generation, key efforts include forecasting renewable energy outputs using weather data and designing advanced power converters. In transmission, the focus is on reducing operational costs while maintaining system stability, such as minimizing voltage fluctuations. In power consumption, better modeling and prediction of energy demand have become crucial to align consumption patterns with the intermittent and variable nature of renewable supply, ensuring more effective grid integration. *Research Challenges.* Several challenges hinder power system research. First, ensuring high reliability requires interpretable models with robust worst-case performance, limiting the adoption of complex machine learning techniques like neural networks. Second, large-scale grid operations involve non-linear and non-convex optimization, leading to significant computational costs that restrict real-time applications. Lastly, practical deployment is complicated by limited access to high-quality operational data due to privacy concerns, creating a domain gap between model training and real-world implementation.

4.1.2. Physical laws in urban power system

The laws of electrical circuits are the most fundamental laws in power systems, such as Kirchhoff's laws and Ohm's laws. These foundational principles lead to more advanced concepts, including power flow analysis and power system simulation, which are critical for understanding and optimizing the operation of modern grids. In addition, characteristics of specific applications introduce more constraints, such as the temporal power demand patterns of electric vehicles and the operational dynamics of water pumps.

With the integration of renewable energy sources into power systems, new physical considerations have become increasingly important. As shown in Table 4, fluid mechanics plays a pivotal role in analyzing the aerodynamic performance of wind turbines, while solid-state physics is essential for optimizing the efficiency of solar panels. These advancements not only add complexity to power system modeling but

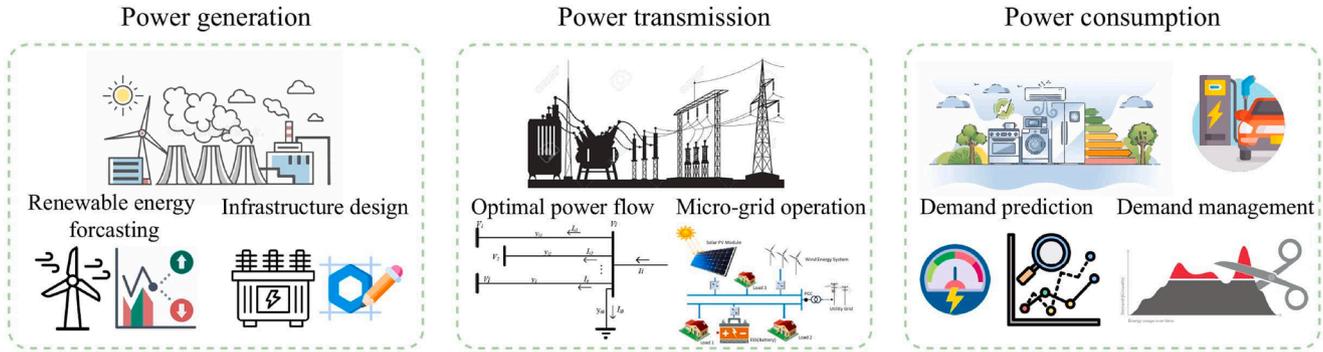


Fig. 7. The figure illustrates the PINN applications in urban energy systems, which are further categorized into power generation, transmission, and consumption.

Table 4
Summary of representative papers on energy systems.

Domain	Paper	Physical model	Physical Theory type	PINN Methods
Power generation	[76]	Power generation principle of solar and wind power.	Physical models	Method 4
	[77]	3-D Navier-Stokes equations.	Physical equations	Method 1 & 6
	[78]	Power converter designing constraints.	Physical models	Method 1 & 3.
Power transmission	[70]	Electrical circuits theory.	Physical equations	Method 1
	[79]	Electrical circuits theory.	Physical equations	Method 1 & 5
	[80]	Simulation of distributed grids.	Expert knowledge	Method 2
	[81]	Operation rules of distributed grids.	Rule-based constraints	Method 1
Power consumption	[82]	Relationship between electricity price and demand.	Rule-based constraints	Method 4
	[83]	2R2C model in building thermal modeling.	Physical models	Method 1 & 7
	[84]	Navier-Stokes equations in pipes.	Physical equations	Method 1

also introduce new challenges and opportunities for the application of PINN in power systems research.

Power Generation. PIML enhances renewable power generation tasks such as infrastructure design, wind field reconstruction, and generation forecasting by embedding physical principles into machine learning models. For example, co-located wind-solar generation prediction benefits from transforming environmental factors (e.g., temperature, humidity) into physically meaningful coefficients like radiation intensity [76], improving model accuracy. To address sparse wind field observations, 3-D Navier-Stokes equations are integrated into model loss functions, enabling physically consistent spatiotemporal wind field reconstruction [77]. In power electronics, PIML guides automated power converter design through a physics-informed surrogate model with a hierarchical architecture, achieving high accuracy with limited training data [78].

Power Transmission. Power transmission focuses on optimizing energy flow from generators to consumers while maintaining grid stability. A key challenge is the non-convex AC-OPF problem, where a neural network approach incorporating Kirchhoff's and Ohm's laws achieves significant computational acceleration-up to 10^4 times faster-via dual Lagrangian relaxation [70], with further validation demonstrating robustness under worst-case scenarios [79]. Beyond OPF, PINN-based multi-agent reinforcement learning (MARL) improves voltage control by introducing auxiliary voltage prediction tasks [80], while federated MARL with physics-informed rewards promotes local energy utilization and enhances micro-grid self-sufficiency [81].

Power Consumption. In the power consumption stage, accurate demand prediction and coordination are essential for reducing grid operation costs, especially with the growing reliance on renewable energy. Aligning fluctuating demand with variable power generation requires incorporating physical laws that govern specific consumption patterns.

For instance, PINN improves city-wide electric vehicle (EV) charging demand forecasting by integrating key prior knowledge, such as the geographical proximity of charging stations and the inverse relationship between price and demand [82]. A pre-training strategy is used, where the model first learns from synthetic data based on these principles

before adapting to real-world data, enhancing prediction accuracy and capturing complex price-demand dynamics. In building energy management, PINN leverages the 2R2C thermodynamic model to predict heating energy consumption [83]. By embedding this model into the loss function, PINN achieves high accuracy in forecasting temperature and air-conditioning power, particularly for long-term predictions. For coordination tasks, PINN aids in optimizing urban water pump operations [84]. A multi-agent reinforcement learning (MARL) framework, trained with a physics-informed surrogate model, predicts pipeline pressure and system-wide energy costs. The Navier-Stokes equations are embedded in the loss function, improving prediction accuracy and enhancing cost efficiency.

4.2. Environment

4.2.1. Research focus and challenges in urban environmental systems

Research on urban environmental systems encompasses five key domains: air quality, water resource management, soil dynamics, waste management, and carbon mitigation [85–87], as shown in Fig. 8. These studies aim to model the physical, chemical, and biological processes shaping urban environments, providing scientific foundations for sustainable development and informed governance.

Research Focus. Air quality research focuses on pollutant diffusion, transport, and chemical reactions to guide mitigation. Water management addresses surface runoff, groundwater flow, and urban water cycles for efficiency and flood control. Soil dynamics investigates water transport, thermodynamics, and consolidation under natural and anthropogenic influences, supporting agriculture and disaster prevention. Waste management models degradation and resource recycling to optimize sustainability.

Research Challenges. Challenges include expressing complex, nonlinear, and multi-scale physical laws within AI models, addressing data sparsity and noise from limited monitoring, and integrating heterogeneous data effectively. Ensuring physically consistent outputs remains critical, particularly in air and water applications where both accuracy and adherence to physical principles are essential.

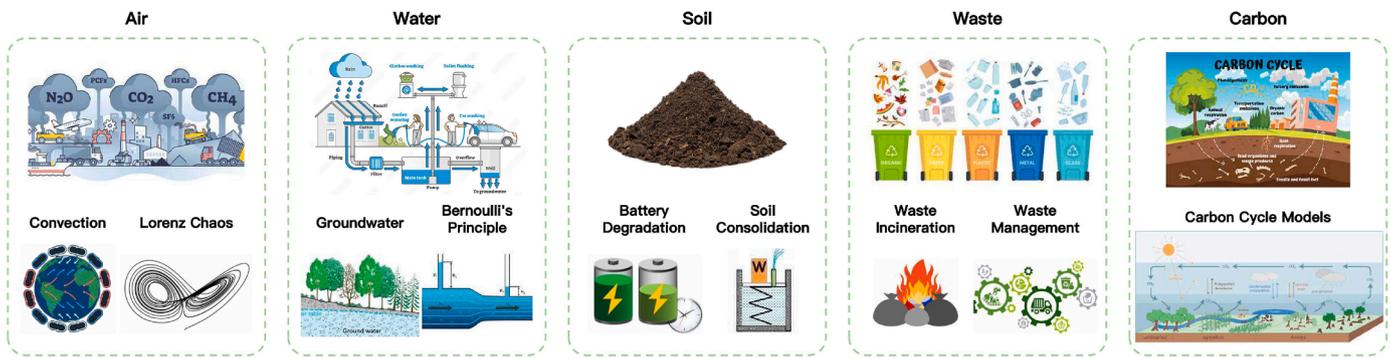


Fig. 8. The figure categorizes environmental applications into Air, Water, Soil, Waste, and Carbon, illustrating key processes and their associated physical mechanisms in each domain.

Table 5
Summary of representative papers on environment systems.

Domain	Paper	Physical Model	Physical Theory Type	Fusion Method
Air	[85]	Diffusion-Advection PDE	Physical equations	Method 6
	[46]	Boundary-Aware Diffusion-Advection	Physical equations	Method 5
	[93]	Finite Volume and PDE Models	Physical equations	Method 6
	[94]	Fractional Lorenz Chaos Model	Physical models	Method 4
	[95]	Pollutant Advection-Diffusion PDE	Physical equations	Method 1
	[90]	Navier-Stokes and Continuity	Rule-based constraints; Physical equations	Method 1
	[96]	Mobility Dynamics	Physical models	Method 7
Water	[86]	HBV Groundwater Dynamics	Physical models	Method 4
	[97]	Richardson-Richards Hydrodynamics	Physical equations	Method 1 & 6
	[92]	Surface Runoff Advection-Diffusion	Physical models; Data	Method 4
	[89]	Flow Conservation and Pressure Relations	Physical laws	Method 4
	[98]	Mass Dynamics in Shallow Water Equations	Physical equations	Method 3
	[55]	Hydro-Mechanical PDEs	Physical equations	Method 5
	[99]	Nonlinear Shallow Water Dynamics	Physical models	Method 1
	[88]	Enhanced Shallow Water Models	Rule-based constraints	Method 1
Soil	[100]	Richards Hydrodynamics	Physical models	Method 5
	[101]	Kinematic, CO ₂ , and Heat Equations	Physical equations	Method 6
	[102]	Solid Waste System Dynamics	Physical models	Method 6
	[103]	Battery Degradation Models	Physical models	Method 6
	[55]	2D Soil Consolidation PDE	Physical models	Method 6
	[104]	Pile-Soil Dynamics Models	Physical models	Method 1
	[105]	Richards Soil Hydrodynamics	Physical equations	Method 5
Waste	[106]	Terzaghi Consolidation PDE	Physical models	Method 1 & 6
	[91]	Radar Backscatter Dynamics	Expert knowledge	Method 2
	[107]	Vegetation-Soil Combustion Physics	Physical equations	Method 4
Carbon	[108]	Ecosystem Productivity Models	Physical models	Method 3
	[87]	Chemical Transport Model	Physical equations	Method 5

4.2.2. Physical theories in urban environmental systems

Urban environmental modeling leverages six categories of physical knowledge-rule-based constraints, physical laws, physical equations, physical models, expert knowledge, and data-as summarized in Table 5. Rule-based constraints ensure model rationality, such as pollutant continuity and terrain-aware hydrodynamics [88]. Physical laws provide deterministic frameworks, including flow conservation, pressure-flow relations, CO₂ balance, and heat conduction [89]. Physical equations, typically expressed as partial differential equations (PDEs), capture pollutant transport, hydrodynamic flows, and soil or waste dynamics [90]. System-level physical models, such as fractional-order Lorenz systems for air dynamics, HBV hydrological models, and Richards' and Terzaghi's theories for soil behavior, offer comprehensive representations [55]. Expert knowledge complements these with domain-specific insights, particularly in waste management [91]. Additionally, synthetic data generated from physical models mitigates observational limitations and enhances AI training, particularly in data-scarce domains such as water resources [92].

Air. Physics-AI integration in air quality research combines physical laws with AI to efficiently model and predict complex pollutant transport. The diffusion-advection equation (DE) was integrated into AirPhyNet, improving prediction accuracy for sparse data and sudden changes [85].

The boundary-aware diffusion-advection equation (BA-DAE), combined with neural ODEs, achieved precise multi-scale dynamic predictions [46]. Finite volume methods (FV) and PDEs, incorporated into PINNs, optimized traffic flow allocation to enhance urban air quality prediction [93]. The fractional-order Lorenz system, integrated into the SARFIMA-NARX framework, captured nonlinear and chaotic dynamics, reducing RMSE by 20% [94]. Pollutant convection-diffusion PDEs, combined with deep graph neural networks (DGM), enhanced PM_{2.5} prediction accuracy [95]. Navier-Stokes equations, integrated with PINNs, improved indoor airflow modeling, reducing errors while enhancing computational efficiency and physical consistency [90]. Finally, a Smart City integrated dashboard systematically combined AI techniques with physical urban models to optimize governance, mobility, and energy strategies, significantly reducing urban air pollution and improving overall sustainability.

Water. Physics-AI integration in water resource research combines physical laws and data-driven techniques for efficient hydrological modeling and accurate predictions. The HBV hydrological model integrated with a causally constrained neural network (H-HBV) improved groundwater level prediction reliability [86]. The Richardson-Richards Equation (RRE), integrated into PINNs, effectively modeled unsaturated soil water flow and solved inverse problems with high efficiency [97].

A deep neural network (DNN) based on surface runoff dynamics improved runoff and pollution control predictions [92]. PI-GNN, incorporating flow conservation and pressure-flow models, optimized large-scale water distribution systems [89]. The shallow water equations, integrated into PT-LSTM, enforced mass conservation and enhanced pollutant transport predictions [98]. PI-MR-NN reduced computational costs while improving hydro-mechanical modeling accuracy [55]. PINNs applied to spherical and enhanced shallow water equations improved flow simulation efficiency [99]. Finally, the PIDL framework with the Richards equation addressed data scarcity in unsaturated infiltration modeling [88].

Soil. Physics-AI integration methods in soil research combine physical equations with data-driven techniques to achieve efficient modeling and accurate predictions [102]. The partial differential equation (PDE) for two-dimensional soil consolidation, Euler-Bernoulli beam theory, and Richards equation were incorporated into Physics-Informed Neural Networks (PINN), significantly improving computational efficiency and accuracy for complex problems such as consolidation and hydraulic parameter inversion [55]. The I2EM model integrated with PIML-FFNN achieved high-precision soil moisture estimation under sparse data conditions [104]. The vegetation-soil combustion model enhanced the generalization ability for post-fire soil severity assessments [105]. The heat transfer equation embedded in PINN with physical constraints improved soil temperature modeling efficiency [106].

Waste. Physics-AI integration methods in waste research combine physical knowledge with data-driven techniques to enhance system modeling and prediction capabilities. Domain knowledge of solid waste management systems was integrated into a hybrid neural network (HNN), improving modeling performance and interpretability under data-scarce conditions to support waste management decisions [91]. Thermodynamic and dynamic models of battery degradation were incorporated into a physics-informed machine learning framework, enabling non-destructive decoupling of degradation modes and lifecycle prediction. Compared to traditional methods, the model achieved a 25-fold increase in prediction speed and 95.1 % accuracy, significantly reducing validation costs [107].

Carbon. PIAI integration methods in carbon emission research combine physical knowledge with data-driven techniques to enhance modeling and prediction. For example, physics-informed deep networks were used to estimate aboveground carbon biomass (AGB) by integrating physical parameters like solar-induced fluorescence (SIF) and gross primary productivity (GPP) into the network architecture, which improved accuracy, particularly in data-scarce regions [108]. Neural Operator models were applied to predict carbon monoxide (CO) concentrations by integrating chemical transport models (CTM) with satellite and ground data. The model enhanced prediction speed and accuracy, especially in extreme pollution events [87]. These methods improve carbon emission modeling, addressing data imbalance and regional bias, with better spatial resolution and temporal consistency compared to traditional methods [109]. However, challenges remain with model robustness and data dependency, which future research should focus on, particularly for global-scale and high-resolution applications [110]. Real-time early warning systems and multi-source data fusion will be key to further enhancing prediction accuracy [108].

4.3. Transportation

4.3.1. Research focus and challenges in urban transportation

Urban transportation research tackles the complexity of modern traffic systems through improved modeling, prediction, and control [111–113]. As shown in Fig. 9, key tasks include traffic state estimation, data imputation, flow prediction, and control. A growing trend is the integration of physics-based models with AI techniques, where PIAI frameworks embed physical laws into neural networks to overcome the limitations of purely data-driven or model-based approaches.

Key challenges include sparse and noisy data, the complexity of non-linear, time-varying traffic dynamics, and the difficulty of incorporating physical constraints into flexible AI models. As shown in Table 6, these challenges can be addressed through various strategies that integrate physics with AI. Additionally, ensuring generalization across regions, scenarios, and conditions requires robust domain adaptation. Addressing these challenges calls for interdisciplinary efforts to advance theory and computation, positioning PIAI as a promising direction for adaptive and sustainable urban mobility [114,115].

Traffic State Estimation. Traffic State Estimation (TSE) aims to infer traffic density, flow, and speed from sparse or noisy sensor data, supporting real-time control and long-term planning. Classical models like LWR [137], CTM [138], and ARZ [139] describe macroscopic traffic dynamics using the Fundamental Diagram and vehicle conservation. PIAI provides a data-efficient solution by embedding physical laws into neural networks. Huang et al. [117] integrated LWR and CTM into deep learning models, achieving accurate, robust estimation under limited observations. Observer-based extensions [119] further improve performance using only boundary data. Shi et al. [122] proposed learning FD parameters jointly with state estimation, extending to ARZ models to capture complex phenomena like stop-and-go waves. Nonlocal models [124] introduce driver anticipation via integro-differential equations, enabling data-driven learning of anticipation kernels. Ka et al. [125] developed the Generalized Bathtub Model to track network-level traffic using time-distance domains. To address uncertainty, PhysGAN-TSE [126], TrafficFlowGAN [127], and SPIDL [128] use generative models and autoencoders to quantify and learn uncertainty distributions. For network-scale TSE, PSTGCN [129] integrates graph convolutions, GRU, and physical constraints, achieving strong performance even with partial data.

Traffic Data Imputation and Calibration. In real-world traffic applications, missing or unreliable sensor data-caused by hardware failures, environmental disturbances, or human error-significantly hinder accurate traffic state estimation and prediction. Thus, inferring missing variables and calibrating model parameters under such imperfect conditions is a key challenge. Tang et al. [73] addressed this by integrating the macroscopic CTM model with an unsupervised denoising autoencoder, enabling robust parameter calibration through minimizing the discrepancy between simulated and observed data. Their framework incorporates boundary conditions (e.g., upstream flow) as inputs, supporting conditional generation and handling data noise effectively without requiring labeled data. Xue et al. [130] combined the Network Macroscopic Fundamental Diagram (NMFd) with Graph Neural Networks (GNNs), enforcing physical consistency in interpolation by embedding traffic flow laws, which improves the accuracy and interpretability of estimated traffic states.

Traffic Flow Prediction. Traffic flow prediction aims to forecast future traffic states using historical and real-time data. While physics-based models offer interpretability and robustness by capturing the underlying traffic dynamics, they often fall short in handling complex patterns. Integrating them with data-driven approaches enhances accuracy, generalization, and computational efficiency. Yuan et al. [131] proposed Gradual Physics Regularized Learning (GPRL), which incrementally embeds traffic flow models-from fundamental diagrams to second-order models like PW and ARZ-into Gaussian processes. This hierarchical integration improves prediction accuracy while significantly reducing computational cost. Ji et al. [132] introduced the Spatio-Temporal Differential Equation Network (STDEN), which models traffic flow as being driven by a latent potential energy field. By embedding this field into neural network architectures, STDEN captures complex dynamics in urban road networks and outperforms existing methods in predictive accuracy. Li et al. [133] developed a two-stage physics-informed transfer learning framework for network-level prediction. Their method leverages Macroscopic Fundamental Diagrams (MFDs) to partition networks and applies Deep Tensor Adaptation Networks (DTAN) for domain adaptation between source and target regions. This approach addresses cold-start and distribution shift problems effectively. Deshpande

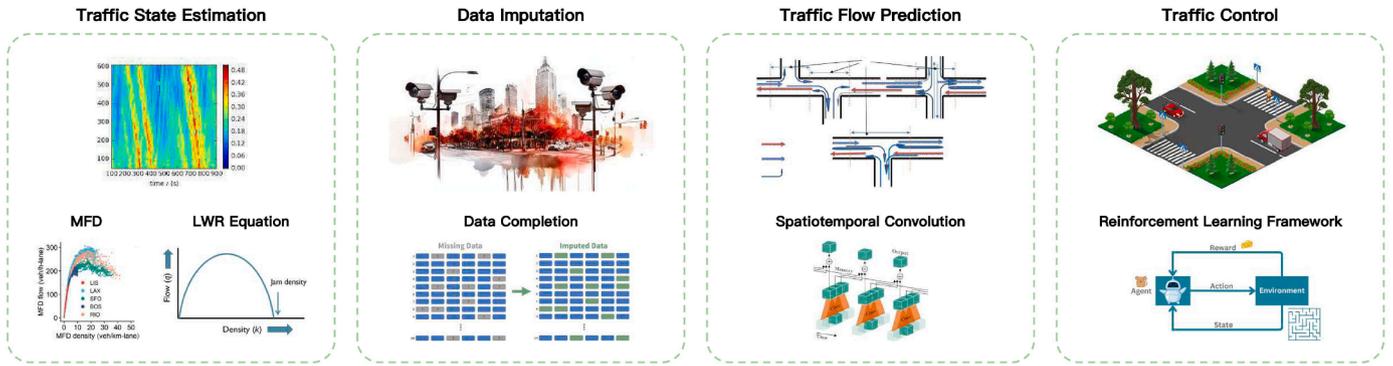


Fig. 9. The figure summarizes four core tasks in urban traffic systems-state estimation, data imputation, flow prediction, and control-highlighting key physical and AI-driven modeling approaches in each.

Table 6
Summary of representative papers on transportation systems.

Research Domain	Paper	Physical Model	Physical Theory Type	Fusion Method
State Estimation	[116,117]	LWR & CTM	Physical models	Method 1 & 4
	[118,119]	ARZ	Physical models	Method 1
	[120-122]	LWR & ARZ	Physical models	Method 1
	[123,124]	non-local LWR model	Physical models	Method 1
	[125]	Generalized bathtub model	Physical models	Method 1
	[126,127]	LWR & ARZ	Physical models, Uncertainty	Method 1 & 4
	[128]	α -SPIDL & β -SPIDL	Physical models	Method 1 & 3
Data Imputation	[73]	CTM	Physical models	Method 1
	[130]	λ -trapezoidal MFD	Physical models	Method 1
Flow Prediction	[131]	LWR, ARZ & CTM	Physical models	Method 1
	[132]	PEF, Differential Equation	Physical equations	Method 1
	[133]	MFD	Physical models	Method 4 & 1
	[134]	LWR	Physical models	Method 7
Control	[135]	Kinematic wave model	Physical models	Method 3 & 4
	[136]	PI-ALINEA	Control strategies	Method 2

and Park [134] proposed a Physics-Informed Graph Convolutional Gated Recurrent Neural Network (PI-GRNN) combined with a Kalman Filter mixture model. This architecture handles non-linearity and uncertainty, while physics constraints ensure that learned dependencies reflect true causal relationships.

Traffic Control. Traffic control focuses on optimizing traffic flow via signals, ramp metering, or other mechanisms, typically formulated as optimization problems. While traditional methods rely on physical modeling, the integration of reinforcement learning (RL) offers greater flexibility and adaptability without sacrificing interpretability. Su et al. [135] proposed an adaptive control system that combines kinematic wave-based traffic models with approximate dynamic programming (ADP). By using parametric value function approximators, their decentralized framework reduces computational burden and enables robust control across large-scale, congested networks. Han et al. [136] developed a physics-informed RL framework for ramp metering, leveraging both real-world and synthetic data to improve policy robustness and reliability. Their hybrid approach outperforms classical feedback and pure RL strategies in reducing travel time and alleviating congestion.

4.4. Economy

4.4.1. Research focus and challenges in urban economics

Urban economics, a critical domain in urban systems research, encompasses production and manufacturing, stock markets, commodity sales, and point-of-interest (POI) recommendations, as shown in Fig. 10. This field aims to address complex dynamics across industrial operations, financial systems, supply chains, and user behavior modeling, emphasizing the integration of physical knowledge and AI for enhanced system optimization and intelligence.

Research Focus. In production and manufacturing, efforts focus on optimizing resource allocation and energy efficiency in energy-intensive industries, combining physical heat conduction models with AI for real-time prediction. Stock market research addresses nonlinear price dynamics influenced by volatility and arbitrage-free principles, supporting risk management and investment strategies. Commodity sales studies optimize supply chains, forecast demand, and manage inventories by integrating physical models and data-driven methods. POI recommendation predicts user behaviors using historical check-ins, accounting for spatiotemporal dynamics and personalized preferences.

Research Challenges. Adhering to Physical Laws: Dynamic processes in urban economics must comply with physical principles, such as heat conduction equations and arbitrage-free conditions, yet embedding these into deep learning remains challenging. Fitting Complex Dynamics: Advanced modeling techniques are required for high-dimensional nonlinear processes, such as stock market volatility, which is described by Heston models. Data Sparsity: Incomplete datasets, such as missing user trajectories in POI recommendations or limited market data during extreme fluctuations, hinder model performance, necessitating synthetic data generation or imputation. Customized Deep Learning Architectures: Tailored models, such as embedding dynamic hedging and arbitrage principles in financial applications, enhance compliance with domain-specific physical laws. Ensuring Stability and Reliability: Models must produce robust outputs under uncertainty, with dynamical constraints improving stability in scenarios like supply chain optimization.

4.4.2. Physical theories in urban economics

Urban economics leverages diverse physical theories to model, analyze, and optimize complex systems. These theories span rule-based constraints, physical laws, equations, models, and expert knowledge,

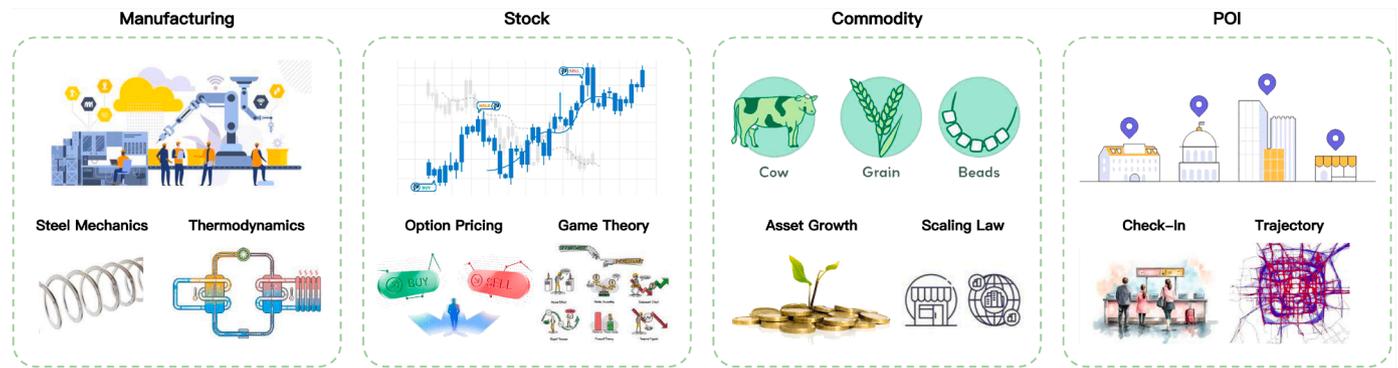


Fig. 10. The figure categorizes economic applications into Manufacturing, Stock, Commodity, and POI, illustrating key processes and their associated mechanisms in each domain.

Table 7
Summary of representative papers on economy systems.

Research Domain	Paper	Physical Model	Physical Theory Type	Fusion Method
Manufacturing	[140]	Heat Transfer Equations	Physical equations	Method 1 & 2
	[141]	Steel Frame Mechanics and Code Constraints	Rule-based constraints, Physical laws	Method 5
	[142]	Thermodynamics and Microstructure Evolution	Expert knowledge	Method 1 & 2
	[143]	Hottel's Furnace Radiation Model	Physical models	Method 1
	[144]	Physical Knowledge in Simulations	Physical laws	Method 1 & 3
	[145]	Structural Reliability Insights	Physical models	Method 1 & 6
	[117]	Industrial Load Physics	Physical laws	Method 1 & 3
Stock	[146]	Option Pricing Models	Physical models	Method 1 & 5
	[71]	Option Pricing Core Physics	Physical models	Method 1 & 4
Commodity	[147]	Corporate Growth Dynamics	Physical models	Method 5
POI	[148]	User Check-In Modeling	Physical equations	Method 7
	[149]	Trajectory Flow Transition Model	Physical models	Method 4

playing critical roles in production and manufacturing, stock markets, commodity sales, and POI recommendations, as shown in Table 7.

Rule-based constraints such as symmetry and non-negativity, underpin physical modeling. In production and manufacturing, design standards ensure compliance with safety and operational norms, forming the foundation for system optimization [141].

Physical laws summarize universal principles derived from empirical observations. In manufacturing, laws like voltage and power relationships guide energy optimization. In structural engineering, forces such as axial, shear, and bending moments ensure mechanical stability [141].

Physical equations often expressed as partial differential equations (PDEs), describe system dynamics. In manufacturing, heat transfer PDEs model thermal processes, while in POI recommendation, graph differential equations capture user behavior's spatiotemporal evolution [140].

Physical models represent complex systems by capturing their multi-scale dynamics. In manufacturing, the Hottel zone method is used to model heat transfer, while reliability analysis assesses structural safety. In stock markets, the Black-Scholes equation characterizes option pricing dynamics. In commodity sales, scaling laws illustrate company growth, and for POI recommendation, trajectory flow graphs describe user transitions between locations [145].

Expert knowledge integrates domain-specific observations to enhance physical modeling. In manufacturing, thermodynamics and material behavior insights drive advanced material design and optimization [142].

Manufacturing. PIAI has revolutionized production and manufacturing by integrating physical theories to address complex challenges efficiently and accurately. Heat transfer problems are solved using PINNs, achieving high-accuracy predictions under uncertain boundary conditions while outperforming finite element methods [140]. FrameRL com-

bines finite element analysis with reinforcement learning, automating steel structure design with sub-second computation times [141]. The material design leverages thermodynamic principles, enabling efficient discovery of high-entropy alloys with limited data [142]. Similarly, the Hottel zone method integrated into PINNs improves furnace heat transfer predictions under sparse data [143]. A hybrid physics-data model (HPDM) enhances interpretability and responsiveness in intelligent manufacturing, reducing costs and increasing automation [144]. Structural reliability is advanced with PINNC, achieving superior accuracy in nonlinear problems [145]. Finally, the PITA model improves industrial load monitoring, enhancing energy decomposition and efficiency management [117]. These innovations demonstrate the transformative impact of PIAI on industrial processes.

Stock. PIAI has significantly advanced option pricing and risk management. A PINN framework integrates the Black-Scholes equation and free boundary conditions, solving high-dimensional, path-dependent problems efficiently and accurately [146]. The FINN framework combines no-arbitrage principles and delta-gamma hedging with neural networks, enhancing computational efficiency and robustness [71]. These approaches provide precise, scalable solutions for complex financial challenges.

Commodity. PIAI enhances growth prediction in commodity sales by integrating company growth models (scaling laws and asset dynamics) with LSTM time-series models. This approach captures long-term trends and short-term fluctuations, significantly improving forecasting accuracy and interpretability, and offering a robust solution for company growth prediction [147].

POI. PIAI improves Next-POI recommendation by addressing data sparsity and behavior modeling challenges. One approach integrates graph differential equations with Siamese networks, embedding user interest dynamics into time-series graphs to enhance accuracy in sparse, long-

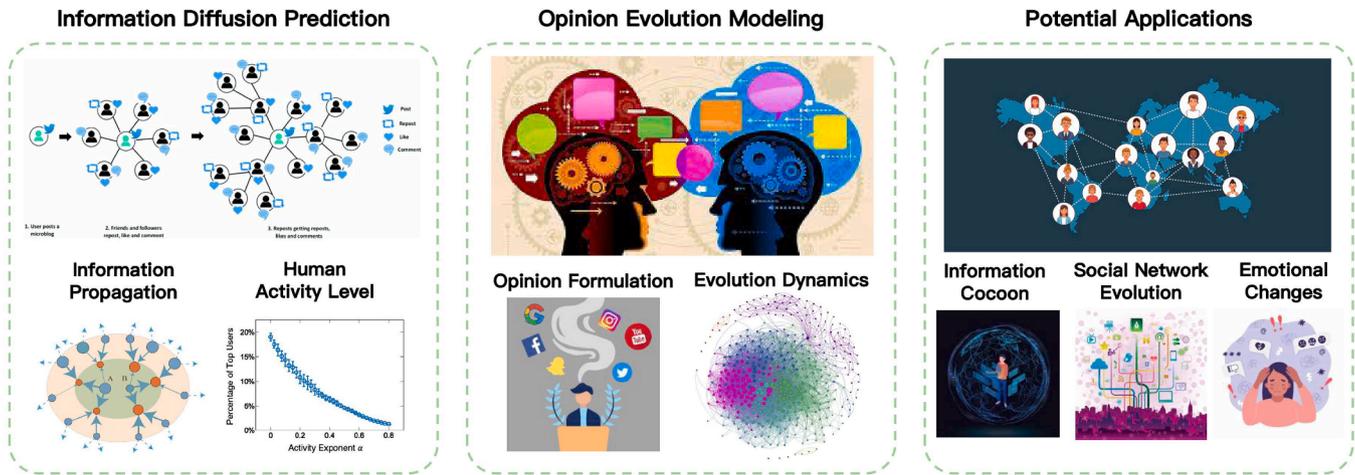


Fig. 11. Applications of PIAI in information systems. The figure illustrates three representative research themes. *Left:* Information diffusion prediction, where PIAI integrates human activity data and cascade dynamics with physics-based diffusion models (e.g., epidemic or point-process priors) to forecast information propagation and source localization. *Middle:* Opinion evolution modeling, which embeds social interaction and external stimuli into physics-inspired coupling mechanisms (such as consensus dynamics and bounded-confidence models) to capture polarization and collective behavioral shifts. *Right:* Potential applications derived from these models, including measuring and mitigating information cocoons, modeling the co-evolution of social networks, and predicting emotion dynamics under external events. Together, these examples demonstrate how PIAI enables interpretable and mechanism-based understanding of complex information behaviors in digital ecosystems.

span scenarios [148]. Another combines trajectory flow graphs with self-attention and uncertainty modeling, leveraging missing POI insertion and confidence calibration to boost recommendation accuracy and robustness [149].

4.5. Information

4.5.1. Research focus and challenges in urban information systems

With the development of digitalization, social networks have become an important part of our daily lives. Information in social networks refers to the various forms of data generated through platforms, which often reflect the structure and dynamics of the social network [150]. Modeling the dynamics of information on social networks provides insight into information dissemination mechanisms, such as how messages spread across networks and what factors influence the prevalence of information. Secondly, user experience can be optimized by analyzing user behavior patterns. For example, a personalized recommendation system can provide more accurate content and services according to users' interests. Moreover, social network modeling can help identify community structures and social influencers, which is crucial for marketing strategy development. An illustration of this part is shown in Fig. 11.

In recent years, with the rapid development of physic-informed machine learning (PIML), researchers have begun to explore new ways to integrate it with the modeling of information dissemination on social networks. This combination not only inherits the accuracy of traditional physics models but also makes full use of the powerful data processing power of machine learning, providing a new perspective for understanding the complex dynamics of information on social networks [151]. In the following part, we will explore the applications of physic-informed AI in information diffusion (also referred to as information propagation and information cascade) prediction and opinion evolution. Then, we will demonstrate the potential applications of physic-informed AI in other scenarios in the information system such as predicting information cocoons and filtering bubbles.

4.5.2. Physic-informed AI in information diffusion prediction

Information diffusion is the process by which information is passed on a social network from one or more sources to other members through the social network platform. With the development of deep learning technology, more and more researchers have begun to use data-driven

methods to predict the spread of information on social networks [160]. However, existing methods have proven not to model the dynamics of information propagation on social networks well, especially phase transitions. Xie et al. [161] model the spread mode of information on social networks by using the frequency of tweets posted by users and their degree on social networks to represent the level of user activity.

Although user activity levels have been shown to be effective in modeling the spread of information on social networks, existing research has not taken this physical law into account [152]. Tang et al. [152] proposed MSA-Net, a multi-scale information diffusion model that can sense user activity level. Specifically, MSA-Net learns three different levels of network representation (micro, meso, and macro) and introduces the concept of user activity level, which measures the difference between individual users by their connectivity and average number of tweets per unit of time. Tu et al. [153] proposed a predictive model of information Diffusion called ODID (Optimized Diffusion Intensity Dynamics). The authors propose a mathematical framework based on the heat transfer equation to model information density flow in social networks, adjusting the information transfer coefficient α to fit actual data. Cheng et al. [154] proposed a new framework called CasDO to solve the problem of cascaded popularity prediction in social network content diffusion analysis. It uses the probabilistic diffusion model and ODEs to deal with the temporal irregularity of information cascade events and the inherent uncertainty of information diffusion. Yu et al. [155] proposed PiGCN which combines structural features with temporal features and introduces the concept of physical information neural networks (PINN) to capture dynamic changes in information propagation.

4.5.3. Physic-informed AI in opinion evolution modeling

Opinion evolution refers to the process by which users' opinions change over time in a social network [162]. In this process, the positive and negative opinions of users interact with each other and together affect the overall opinion dynamics. Studying the evolution of opinion is helpful in understanding the formation and development mechanism of public opinion, and is of great significance for predicting the change in public attitude, evaluating the effect of policies, and coping with social events. Traditional opinion evolution methods typically model opinions dynamically as discrete and homogeneous processes, meaning that they update user opinions at fixed intervals and assume that all users or nodes follow the same evolution rules [159]. However, these assump-

Table 8
Summary of representative papers on information systems.

Research Domain	Paper	Physical Model	Physical Theory Type	Fusion Method
Information diffusion	[152]	User Activity Level Equations	Expert knowledge	Method 7
	[153]	Heat Transfer Equation	Physical laws	Method 6
	[154]	Probabilistic Diffusion Equations	Physical Equations	Method 6
	[155]	Probabilistic Diffusion Model	Physical Equations	Method 1
Opinion Evolution Modeling	[156]	Jump-Diffusion Stochastic Differential Equations	Physical Equations	Method 6
	[157]	Hegselmann-Krause Model	Physical models	Method 6
	[158]	Neural ODEs and Higher Derivative Constraint	Physical Equations	Method 6
	[159]	Bidirectional Graph ODEs	Physical models	Method 5

tions make the modeling of opinion evolution suboptimal. Therefore, we need to introduce physic-informed AI to model the heterogeneity among users and combine physical laws to realize the modeling and prediction of opinion evolution.

De et al. [156] proposes SLANT, a probabilistic framework for modeling and predicting opinion dynamics in social networks. It utilizes labeled jump-diffusion stochastic differential equations to represent changes in user opinions over time and allows efficient simulation of models and estimation of parameters from historical event data. Okawa et al. [157] proposed SINN, which combines large-scale social media data with prior scientific knowledge from sociology and social psychology by transforming traditional opinion dynamics models into ordinary differential equations (ODEs) and approximating these ODEs using neural networks. Li et al. [158] proposed a method called HiDeS NODE. It extends the expressive range of NODE and enhances the predictive capability by introducing the interaction between higher-order derivatives and state variables and using state vectors and their higher-order derivatives as supervisory signals in training. Duan et al. [159] proposed a framework called Bi-Dynamic Graph Ordinary Differential Equation (BDG-ODE) to simulate the dynamic changes of opinions in social networks. It captures the complexity of opinion evolution through two dynamic processes: the evolution of positive and negative opinions. The model includes a dual opinion encoder that handles both positive and negative opinions separately and models the evolution of opinions over time through the bidirectional graph ordinary differential equation, enabling continuous capture of opinion changes. In addition, an opinion synthesis decoder is introduced, which can effectively map the representation evolved from the potential space back to the opinion space.

4.5.4. Potential applications of physic-informed AI in information systems

In addition to the typical research problems mentioned above, there are a large number of problems in the information space that can be solved with the help of physic-informed AI. Such as information cocoon modeling [163], relationship evolution in social networks [164], emotional and psychological state changes [165], etc. Existing methods have used dynamic equations to describe the evolution of relevant system dynamics, but how to use physic-informed AI to enhance the accuracy and efficiency of modeling is still an open question.

4.6. Public services

4.6.1. Research focus in urban public services.

Urban public services play a crucial role in the urban system by providing essential services that maintain residents' health, safety, and social well-being. *Public health* focus on evaluating and forecasting the overall health status of residents, thus ensuring access to fundamental healthcare and safeguarding the right to health and corresponding subsidies for all citizens. *Safety and social security* aim to accurately predict the movements of urban populations, including crowds and traffics, ensuring public safety and offering support to vulnerable populations. *Service supply and planning* emphasize the provision of cultural, recreational, education, dining, and other community infrastructure, ensuring that urban residents have access to adequate services and facilities to enrich

their daily lives. Together, these elements create a cohesive framework that supports the overall functionality and resilience of urban systems.

Research Challenges. Predicting and preventing epidemic outbreaks is a key challenge in urban public health. The COVID-19 pandemic emphasized the need for effective forecasting models, as urban disease spread is influenced by social behavior, mobility, and healthcare infrastructure. Integrating diverse data sources and generalizing across urban contexts complicates this task. Approaches like PINNs and EINNs (Epidemiology-Informed Neural Networks) combine physical principles with data-driven techniques, but issues such as parameter identifiability, uncertainty quantification, and real-time adaptability remain. Urban safety and social security research faces challenges in predicting crowd dynamics and vehicle movements in rapidly urbanizing environments. Traditional models struggle with the heterogeneity of human behavior and the interactions between pedestrians, vehicles, and infrastructure. To address these complexities, advanced approaches like PIAI and machine learning are essential for improving predictive accuracy and real-time decision-making in urban safety management. Urban service supply and planning involve the complex interplay between urban structures, population dynamics, and the accessibility of essential services, necessitating resilient and adaptive solutions. Traditional optimization methods often struggle with the nonlinear and dynamic nature of urban systems, leading to high computational demands and less effective outcomes.

4.6.2. Physical theories in urban public services.

The study of urban public health mainly relies on physical models (often expressed as ODEs) that characterize the spread of epidemics in urban populations with various variants, such as the SIR (Susceptible - Infected - Recovered) model and the SEIR (Susceptible - Exposed - Infected - Recovered) model [176]. Recent extensions of these models take into account factors such as social behavior, mobility patterns, and functional-order dynamics.

To support the prediction of crowds and road traffic for public safety, it is essential to employ physical theories that can effectively characterize both individual movements and their macro-level patterns. One key physical model is the social force model [14], which explains how social interactions and pedestrian behaviors are influenced by societal norms, pressures, and the dynamics of group behavior. Another area of research applies physical laws from fields such as fluid dynamics and electromagnetism to accurately characterize individual movements.

Key physical theories in urban service supply and planning include transportation flow, energy exchange in microclimates, and the impact of environmental hazards like floods and earthquakes. These models are crucial for designing resilient infrastructure to withstand urban hazards and climate change.

Public health. Recent research efforts have demonstrated the power of integrating physics theories with AI techniques to improve epidemic modeling and forecasting. Kharazmi et al. [166] extend the classical SIR model with PINNs, using neural networks to infer time-dependent parameters and unobserved dynamics, thus enhancing the accuracy of COVID-19 spread predictions across diverse regions. Rodriguez et al. [167] introduce Epidemiology-Informed Neural Networks (EINNs), a novel framework that merges traditional mechanistic models with AI's

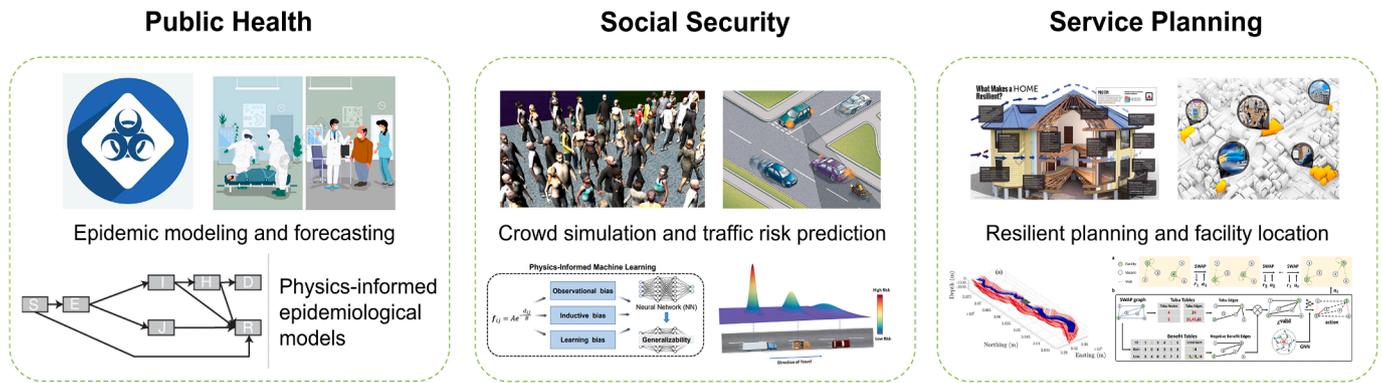


Fig. 12. The applications of PIAI in urban public service scenario.

Table 9
Summary of representative papers on public service systems.

Domain	Paper	Physical Model	Physical Theory Type	Fusion Method
Public Health	[166]	Fractional-order epidemiological models	Physical models	Method 6
	[167]	SEIRM and SIRS model	Physical models	Method 1
	[168]	Compartmental epidemiological models	Physical models	Method 2
	[169]	SICRD compartmental model	Physical models	Method 3
	[74]	Multi-scale SIS model	Physical equations	Method 3
Safety and social security	[55]	Social force model	Physical equations	Method 7
	[170]	Conservation of mass principle	Physical laws	Method 4
	[171]	Road user safety field model	Physical equations	Method 4
Service supply and planning	[172]	Intelligent driver model; Social force model	Physical equations	Method 3
	[173]	Natural hazard models	Physical models	Method 4
	[174]	Four urban weather models	Physical models	Method 4
	[175]	Facility location knowledge	Expert knowledge	Method 2

capacity to process diverse data sources, addressing challenges in both short-term and long-term epidemic forecasting. This approach allows EINNs to learn latent dynamics while efficiently handling heterogeneous data, improving the adaptability and accuracy of predictions. Millevoi et al. [168] employ a reduced-split approach using PINNs, which tracks temporal changes in parameters like transmission rates and state variables, achieving enhanced forecasting accuracy and computational efficiency compared to traditional joint training methods. Hu et al. [169] apply PINNs to the Susceptible-Infected-Confirmed-Recovered-Deceased (SICRD) model, focusing on estimating unknown infected compartments and parameters by introducing wavelet transforms for data preprocessing and modifying the loss function to handle multiple unknowns, significantly improving prediction reliability. Tang et al. [74] introduce the Multi-scale Spatial Disease prediction Network (MSDNet), which combines macroscopic population flow data with microscopic contact patterns, effectively capturing disease spread dynamics while incorporating mobility factors like travel restrictions. This hybrid approach leads to a 15–30% improvement in prediction accuracy over existing models. These studies highlight the fusion of physical models with AI techniques, enabling more accurate, adaptable, and efficient epidemic forecasting in urban environments, while offering flexible tools that can adapt to the dynamic nature of disease spread. By combining physical laws with machine learning, these efforts set a new standard for epidemic modeling, providing critical insights for real-time public health interventions.

Safety and social security. Recent studies highlight the effectiveness of combining physics theories with AI techniques to tackle complex urban safety challenges. Zhang et al. [55] introduce a PIML framework that combines traditional physics-based models with neural networks to enhance the accuracy of crowd simulation and dynamics, offering more reliable predictions for urban safety. Li et al. [170] apply PINNs to predict crowd density by incorporating fluid dynamics principles, significantly improving the efficiency and precision of crowd movement models. Arun et al. [171] integrate safety field theory, inspired by electromagnetic fields, with AI techniques to estimate crash risk and sever-

ity, providing a more context-sensitive and accurate approach to traffic safety. Lee et al. [172] leverage a data-driven PINN framework to predict vehicle and pedestrian trajectories, using Monte Carlo simulations to assess potential collisions and enhance preemptive risk mitigation. These studies demonstrate how combining physical laws with machine learning enhances model accuracy and generalizability, offering more effective urban safety management strategies. The fusion of AI with physical models allows for improved handling of dynamic interactions between pedestrians, vehicles, and infrastructure, addressing limitations of traditional methods. The integration of data-driven insights with physical modeling provides robust tools for real-time risk evaluation and mitigation. Furthermore, these innovations showcase the potential of hybrid approaches to enhance predictive capabilities across diverse urban scenarios. Ultimately, the application of PIAI offers transformative solutions for crowd safety, traffic management, and broader urban social security.

Service supply and planning. Recent research in urban service supply and planning highlights the integration of physics theories with AI techniques to address complex urban challenges. Jenkins et al. [173] fuse high-resolution physics-based hazard simulations with AI to predict the impact of climate change and multi-hazard events, providing critical insights for resilient urban planning. Mao et al. [174] develop the Urban Weather Generator (UWG), which combines physics-based microclimate simulations with AI to optimize energy efficiency and sustainability in urban buildings. Su et al. [175] propose a PIAI approach to solving the Facility Location Problem by using graph theory and reinforcement learning to optimize facility placement and maximize accessibility with reduced computational costs. These studies demonstrate how the fusion of physical models, like transportation flow, climate interactions, and hazard simulations, with AI methods can provide adaptive, scalable solutions for urban planning. The integration of physics-based knowledge with AI enhances the predictive accuracy and efficiency of urban service allocation, improving the overall resilience of cities. By leveraging domain-specific physical laws, these models offer valuable decision-

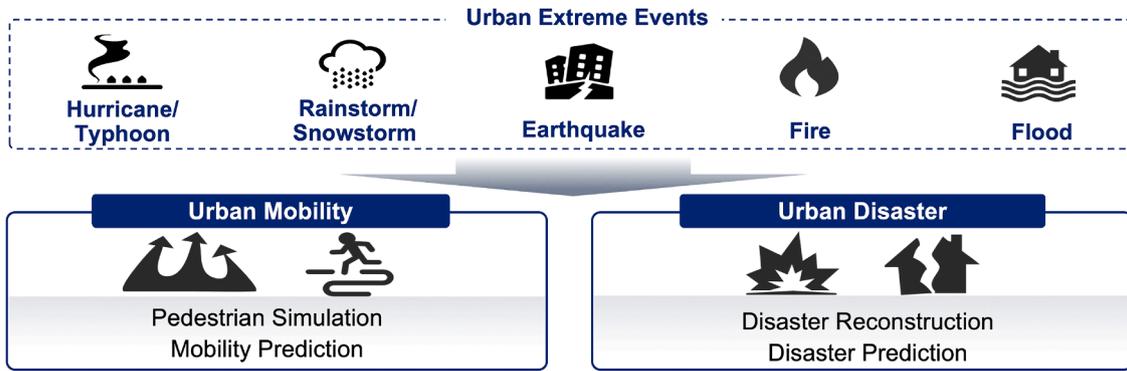


Fig. 13. The applications of PIAI in emergency management.

Table 10
Summary of representative papers on emergency management systems.

Research Domain	Paper	Physical Model	Physical Theory Type	Fusion Method
Mobility Simulation and Prediction	[170]	Spatio-temporal Decay Model	Physical Equations	Method 7
	[55]	Social Force	Physical Equations	Method 4
	[177]	Social Force	Physical Equations	Method 7
Disaster Modeling and Prediction	[75]	HEC-RAS 2D model	Physical Models	Method 1
	[178]	Delay-embedding Theorem	Physical Models	Method 3
	[179]	Fault elastic coupling and stiffness model	Physical Constraints	Method 1
	[180]	Multi-scale representation	Expert Knowledge	Method 2
	[181]	Dislocation model	Physical Equations	Method 1
	[182]	Advective conservation and convective initiation dynamics	Physical Models	Method 7
	[183]	Level-set equation	Physical Equations	Method 1
	[184]	WRF-SFIRE wildfire simulator	Physical Models	Method 7
	[185]	Reacting Navier-Stokes equations	Physical Equations	Method 1
[186]	WRF-SFIRE wildfire simulator	Physical Models	Method 1	

making tools for future-proofing urban infrastructures and promoting sustainable urban development.

4.7. Emergency management

4.7.1. Research focus and challenges in urban emergency management.

As we discuss in Section 3, researchers in the domain of urban emergency management mainly focus on planning, coordinating, and executing strategies to respond to and mitigate disasters like natural calamities, industrial accidents, or other public safety threats, which is referred to as urban extreme events in the following paper. To achieve this goal, numerous methods are developed to sense and predict disaster dynamics and human behavior under the influence of urban extreme events. Therefore, in the following part of this section, we will first introduce the useful physical theories in urban emergency management. Then, we will discuss existing works leveraging PIAI to reconstruct, predict, or simulate human behavior and disaster dynamics.

4.7.2. Physical theories in urban emergency management

Within the domain of urban emergency management, numerous physical theories are utilized to model urban dynamics and human dynamics. In terms of human behavior, a well-known physical theory is the social force model [14,187], which is able to model the microscopic human movement under the influence of urban extreme events and has been discussed detailedly in Section 3. Another famous physical theory describing macroscopic human movement under the influence of urban extreme events is the spatio-temporal decay model [188], which can be represented as $r_i(t) = \bar{r}_i / (1 + k(t) \sum_{j=1}^L w_{ij} N_j(t))$, where \bar{r}_i is the activity of human mobility without the influence of urban extreme events, $r_i(t)$ is the activity of human mobility under the influence of urban extreme events, and $N_j(t)$ is the metric of the spatiotemporal disaster intensity for geographical region j . Furthermore, to predict disaster dynamics, diverse physical theories have been systematically applied to model the distinct physical processes underlying various extreme events. For

instance, fluid dynamics principles are employed for flood prediction, while WRF-SFIRE are employed for wildfire forecasting. We will discuss them in detail in the following sections.

Mobility simulation and prediction for emergency management.

Simulating and Predicting human mobility is crucial for emergency management, as it can effectively support a range of emergency response tasks. In terms of microscopic mobility behavior, pedestrian simulation is instrumental for avoiding incidents like crowd pushing during disasters. In terms of macroscopic mobility behavior, individual mobility or mobility flow prediction are important for applications including location-based early disaster warnings, pre-distribution of rescue resources, and planning for humanitarian aid, etc. Specifically, Li et al. [170] utilize the physical knowledge from the spatiotemporal decay model [188] in the way of Method 6 to predict the post-disaster population flow. Zhang et al. [55] and Chen et al. [177] utilize the physical knowledge from the social force model to simulate crowd dynamics, where physical knowledge are integrated into neural networks in the manner of Methods 3 and 6, respectively. These methods enable a better understanding and prediction of human behavior from micro-to-macro scales, thereby reducing the impact of disasters on urban residents.

Disaster modeling and prediction for emergency management. At the same time, a large portion of the work still focuses on leveraging PIAI methods to better modeling and predicting disaster, thereby enabling more effective urban emergency management. [75] utilize the data generated by the physical model of hydrodynamics to train their AI models to predict peak inundation depths of floods, respectively. [178] draw inspiration from the delay-embedding theorem to detect earthquake precursors from geodetic data. [179] incorporate fault zone physics into predictive deep learning models utilizing PINNs. Tang et al. [180] draw inspiration from the multi-scale characteristics of cascade failures for interdependent infrastructure networks to predict them effectively. [182] integrate the advective conservation and convective initiation dynamics into a neural network architecture for extreme precipitation nowcasting. [183] solve the level-set equation in the way of PINN for wildfire

fire-front modeling. [184] utilize PIAI methods for better wildfire propagation prediction by reimplementing key components of the WRF-SFIRE wildfire simulator by replacing traditional numerical solvers. [185] and [186] utilize PINNs, to reconstruct velocity fields in puffing pool fires and infer wildfire arrival times from satellite data, respectively. Overall, these methods advance the precision and efficiency of disaster modeling across diverse hazards by combining well-established physical knowledge and powerful deep learning models.

4.8. Urban system as a whole

Urban systems are inherently interdependent and dynamic, comprising multiple coupled subsystems. Traditional modeling approaches have largely focused on individual subsystems in isolation, which limits their ability to capture cross-domain interactions and systemic behaviors. Recent research highlights the importance of holistic modeling that integrates these diverse domains, providing a more faithful representation of urban complexity. PIAI offers a promising paradigm for such integrated modeling, as it combines the explanatory power of physical laws with the flexibility of data-driven learning, enabling the joint analysis of heterogeneous urban processes [21].

Cross-domain dynamics and multi-scale integration. The evolution of PIAI for holistic urban modeling has progressed from early efforts targeting individual subsystems to increasingly sophisticated frameworks capable of capturing cross-domain dynamics and multi-scale interactions. Early studies primarily focused on embedding physical constraints into single-domain models, such as traffic flow prediction [116] or building energy modeling [189]. With advances in PIAI methodologies, researchers have shifted toward integrated modeling of interacting urban subsystems. Lin et al. developed a physics-informed graph neural network that couples flood dynamics with traffic modeling, embedding hydrodynamic diffusion equations into a GNN architecture to improve traffic flow prediction during urban flooding events [190]. Li et al. proposed a physics-informed neural ODE model for post-disaster human mobility recovery, incorporating physical movement constraints to better simulate population flow dynamics under disrupted conditions [170]. Tang et al. introduced an integrated graph-based framework for modeling cascading failures across interdependent infrastructure networks-including power, transportation, communication, and buildings-achieving superior accuracy in forecasting systemic disruptions [180].

Recent works further extend PIAI applications to routine urban operations. For example, Rethnam and Thomas combined urban microclimate models with building thermal physics to predict city-wide indoor comfort levels [191], while de Giuli et al. applied physics-informed neural networks for modeling and optimization of district heating systems, providing efficient and physically consistent surrogate models for coupled thermal energy networks [192]. These developments illustrate the expanding scope of PIAI from isolated subsystem modeling to unified treatment of heterogeneous dynamics across domains. PIAI not only supports multi-scale integration-bridging local dynamics (e.g., building-level thermal behavior) with city-wide or regional trends (e.g., energy flows or urban climate patterns)-but also enables multi-source data fusion while maintaining physical consistency [192]. With these capabilities, PIAI is increasingly regarded as a powerful tool for urban system-of-systems modeling, providing enhanced predictive accuracy, interpretability, and generalization for complex urban scenarios [180].

5. Discussions and open problems

5.1. Selecting physical-AI fusion methods for specific tasks

Selecting physical-AI fusion methods in urban computing requires understanding the core strengths and limitations of each paradigm. Physics-based models, grounded in conservation laws, offer strong interpretability and robustness in data-scarce scenarios [193], but struggle with nonlinear complexity and high-dimensional efficiency. AI models,

by contrast, excel in capturing complex patterns and enabling fast inference [194], yet rely heavily on data and often lack physical consistency.

Method selection should consider three key factors: the degree of physical knowledge, data availability, and spatiotemporal scale. Combined with task-specific goals-such as accuracy, efficiency, and interpretability-these dimensions form a decision space for model design. Physics-dominant models suit tasks with strong physical constraints; deep learning with physical validation fits weaker constraints [195]. Hybrid models guided by physical priors are ideal for sparse data, while data-rich settings favor AI models with embedded physical regularization [196]. Real-time tasks benefit from lightweight physics-informed networks; offline tasks allow more complex, tightly coupled architectures. Beyond task-specific strategies, a unified, general-purpose physical-AI foundation model is a promising direction. Current approaches remain largely data-driven; early explorations like UniST and UrbanDiT [197,198] introduce knowledge-guided prompts, yet further efforts are needed to embed physical priors and enhance generalization across diverse urban contexts.

5.2. Limitations and future directions of PIAI in urban systems

Physics-AI fusion methods face several challenges in urban computing [199]. PINNs lack theoretical guarantees for convergence and generalization, limiting their reliability in high-stakes scenarios. Their training is resource-intensive, hindering real-time deployment [120]. Existing models also struggle with multi-scale phenomena like turbulence and chaos, and though physics priors reduce data demands, high-quality data is still needed for calibration. Beyond formal physical laws, urban systems involve rich relational knowledge, such as causality and domain rules-often best represented through knowledge graphs [200,201]. Yet current frameworks rarely incorporate such structured, non-symbolic knowledge, limiting interpretability and reasoning. Scalability remains a bottleneck, constraining applications in large-scale digital twins and real-time simulations.

Future research should strengthen theoretical foundations, improve training efficiency, and explore architectures like multi-resolution networks to handle urban complexity better [132]. Data-efficient strategies-e.g., active and transfer learning-are also essential. Cross-disciplinary collaboration will be key to integrating physics-AI approaches into real-world urban systems. A promising direction is combining PINNs with large language models to enhance expressiveness and enable multimodal reasoning [202-204]. Hybrid architectures and co-training strategies are needed to fully exploit this potential. Looking ahead, developing world models-simulation frameworks that learn, predict, and reason about urban dynamics-offers a path toward interpretable, adaptive, and generalizable digital twins [205]. By fusing physical laws, relational knowledge, and generative learning, world models may serve as the next-generation engines for urban intelligence.

5.3. Key findings on PIAI in urban systems

PIAI methods offer great potential for urban systems, enabled by the structural complexity of cities and the coexistence of physical laws and abundant data. Urban infrastructure encompasses diverse, well-defined physical mechanisms across its core subsystems, while modern sensing technologies provide high-frequency, heterogeneous, and multimodal data. This combination of strong physical priors and rich observational input forms a robust foundation for deploying PIAI models in practical urban scenarios. From a modeling perspective, data-driven strategies often favor AI-dominant approaches to enhance predictive accuracy, especially when dealing with large-scale and noisy datasets. These methods excel in capturing nonlinear dynamics and extracting high-dimensional patterns. In contrast, urban governance and decision-making applications tend to prefer physically grounded models due to their interpretability and alignment with known laws [206]. The dual nature of urban systems-marked by both human-driven uncertainty and physical

regularity, along with complex interdependencies across subsystems—demands a balance between physical interpretability and AI’s representational power. As a result, urban computing emerges as a natural arena for interdisciplinary innovation, where PIAI methods can address critical challenges and drive progress across science and practice.

5.4. Data availability and evaluation standards

To complement the methodological and application discussions, we provide a concise overview of the data foundations and evaluation frameworks that underpin PIAI in urban systems. A comprehensive description is presented in [Appendices E and F](#).

Data foundations and challenges. Urban data form a heterogeneous ecosystem encompassing multiple subsystems such as energy, environment, economy, transportation, information, public services, emergency management, and city-wide integration. These datasets combine long-term statistical records with high-frequency sensing streams, reflecting both physical infrastructure and human dynamics. However, they face persistent challenges including sparse and missing observations, multimodal heterogeneity, privacy and access constraints, and temporal drift caused by evolving urban processes. Recent studies have introduced several mitigation strategies: physics-consistent synthetic augmentation and reconstruction to address data sparsity; physics-constrained multimodal fusion for heterogeneous integration; privacy-preserving federated collaboration to enable cross-institutional learning; and cross-timescale modeling to ensure consistency between short-term perturbations and long-term equilibrium. Representative public datasets and their corresponding applications are summarized in [Appendix E](#).

Evaluation and benchmarking. The assessment of PIAI models requires standardized and multidimensional evaluation criteria that move beyond conventional accuracy-based metrics. An effective benchmarking framework should jointly consider five complementary dimensions: predictive and data-level performance, physical consistency, interpretability and causality, cross-domain generalization and robustness, and system-level impact. Accordingly, the evaluation metrics span from conventional indicators such as RMSE, MAE, and R^2 , to physics-based measures such as conservation error, physics residual, and boundary deviation; interpretability indicators such as symbolic fidelity and causal alignment; and robustness indicators including domain gap, noise stability, and adaptation efficiency. At the system level, indicators such as cross-subsystem consistency, coupling accuracy, and sustainability indices reflect PIAI’s contribution to coordinated and reliable urban governance. Representative benchmark datasets and protocols are provided in [Appendix F](#).

5.5. Open problems

Despite the promising progress of PIAI in urban systems, several critical open problems remain. First, the theoretical foundation of PIAI methods, particularly regarding convergence, generalization, and error bounds in complex, high-dimensional urban environments, remains underdeveloped. Second, there is a pressing need for unified frameworks that can seamlessly integrate heterogeneous physical laws—ranging from deterministic equations to stochastic and agent-based models—into AI architectures for comprehensive urban modeling. Third, current methods lack robustness when faced with noisy, incomplete, or biased urban data, highlighting the necessity for advanced data assimilation, uncertainty quantification, and domain adaptation techniques. Fourth, most PIAI applications remain domain-specific; developing transferable and generalizable models that can adapt across different urban subsystems is an open challenge. Fifth, the integration of symbolic reasoning, causal inference, and structured knowledge representations into PIAI remains limited but crucial for enhancing interpretability and decision support. Lastly, practical deployment of PIAI in real-world urban digital twins is still constrained by scalability, computational cost, and the lack of systematic validation against dynamic, multi-source urban data streams.

Addressing these open problems will be pivotal for advancing the scientific and practical impact of PIAI in complex urban systems. Beyond identifying open problems, we provide concrete research pathways that target (i) theoretical guarantees (convergence, generalization, error bounds), (ii) unified modeling across heterogeneous physical formalisms and scales, (iii) robustness under noisy, missing, and biased data, (iv) cross-domain transferability, (v) interpretability via symbolic and causal structure, and (vi) scalable deployment with dynamic validation. A concise agenda is summarized here, with full details and examples in [Appendix D](#).

6. Conclusion

This study provides a systematic review of PIAI in complex urban systems, establishing a unified taxonomy that spans three major paradigms—AI-dominant, Physics-AI hybrid, and Physics-dominant models—and summarizing seven representative methodological categories. This taxonomy offers a structured understanding and practical guidance for selecting PIAI strategies across diverse urban application scenarios. The results demonstrate that the deep integration of physical knowledge and data-driven intelligence can substantially enhance predictive accuracy, interpretability, and robustness in key urban subsystems, including energy, environment, transportation, economy, information, public services, and emergency management. By combining the rigor of physical laws with the flexibility of AI, PIAI provides a new paradigm for reliable modeling and scientific decision-making in dynamic and multiscale urban processes.

Building upon this foundation, the paper further emphasizes that PIAI is not merely a modeling methodology but also a core technological framework for future urban real-time decision-making and predictive maintenance. By embedding conservation laws, boundary conditions, and uncertainty quantification into intelligent algorithms, PIAI enables interpretable, auditable, and physics-consistent online inference and intervention for tasks such as traffic signal optimization, energy scheduling, pollution response, and infrastructure health monitoring. This transition from prediction to decision marks PIAI as a key enabler for the intelligent, resilient, and sustainable operation of cities.

Finally, we call on the research community to continue advancing PIAI through structured enhancement of convergence and generalization, theoretical characterization of uncertainty and error bounds, unified differentiable coupling across modeling paradigms, physics-consistent robustness under imperfect data conditions, and deeper integration of neural, symbolic, and causal reasoning for interpretability and controllability. At the engineering level, further developments in modular and hierarchical computation, lightweight and online assimilation, automated validation, and uncertainty quantification will be essential to ensure the efficiency, scalability, and reliability of PIAI deployment in real-world urban systems.

CRedit authorship contribution statement

En Xu: Writing – review & editing, Writing – original draft, Investigation, Conceptualization; **Huandong Wang:** Writing – review & editing, Writing – original draft, Investigation, Conceptualization; **Yunke Zhang:** Writing – review & editing, Writing – original draft; **Sibo Li:** Writing – review & editing, Writing – original draft; **Yinzhou Tang:** Writing – review & editing, Writing – original draft; **Zhilun Zhou:** Writing – review & editing, Writing – original draft; **Yuming Lin:** Writing – review & editing, Writing – original draft; **Yuan Yuan:** Writing – review & editing; **Xiaochen Fan:** Writing – review & editing; **Jingtao Ding:** Writing – review & editing, Supervision, Investigation, Conceptualization; **Yong Li:** Writing – review & editing, Supervision, Investigation, Funding acquisition, Conceptualization.

Data availability

No data was used for the research described in the article.

Declaration of competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Extended methodological details

This appendix supplements [Section 2](#) by presenting detailed formulations and examples for the seven representative methods of PIAI. While the main text emphasizes the conceptual taxonomy of three paradigms—Physics-Integrated AI, Physics-AI Hybrid Ensemble, and AI-Integrated Physics—this appendix provides a deeper view into the implementation logic of each method, including its physical rationale, mathematical structure, and representative urban applications.

A.1. PINN with loss function

The core idea of this method is to explicitly incorporate physical constraints (e.g., conservation laws, partial differential equation residuals, or energy balance conditions) into the loss function of neural network training. In this way, the model minimizes both the data-driven error and the physical residual simultaneously, ensuring physical consistency and stable generalization even under limited or noisy data conditions.

A representative example is the PINN proposed by Raissi et al. [28]. This framework uses automatic differentiation (AutoDiff) to compute derivatives of the governing equations and introduces the residual term

$$f_\theta = u_t + \mathcal{N}[u_\theta; \lambda]$$

into the training objective as part of the loss function:

$$\text{MSE} = \text{MSE}_u + \text{MSE}_f,$$

where MSE_f penalizes deviations from the underlying PDE. In this manner, the model achieves unified modeling for both forward problems (solving for state variables given equations) and inverse problems (identifying equation parameters from observations). PINN has been successfully applied to canonical equations such as Burgers [207], Allen-Cahn [208], and Navier-Stokes [209], where the physical residual acts as a structured regularizer, enabling interpretability and transferability even with sparse data.

In urban system applications, this concept can be directly extended to energy and environmental domains: *Energy systems*: In urban power load forecasting, the residual of the power flow equation is added to the loss function to ensure that predicted results satisfy nodal power conservation, thereby maintaining system balance under peak or abnormal conditions. *Environmental systems*: In air quality modeling, the advection–diffusion equation residual is embedded in the PINN framework, enabling the model to maintain physical continuity of pollutant concentration fields even with sparse or discontinuous observations.

This method represents the typical fusion pathway from physical knowledge to loss constraint and belongs to the Physics-Integrated AI paradigm. It extends traditional data-driven models into a joint learning mechanism constrained by both data and equations, enhancing not only predictive accuracy but also physical interpretability and systemic consistency.

A.2. PINN with weight initialization

This method leverages results from physical simulations or analytical models to initialize network parameters prior to training, allowing the learning process to begin from a physically plausible state. By reducing instability from random initialization, it significantly accelerates convergence and ensures that the model adheres to conservation, stability, and dynamical consistency even with limited data.

A representative study is the *Physics-Guided Recurrent Neural Network* (PGRNN) by Jia et al. [33]. In this approach, the *General Lake Model* (GLM) [210] serves as a physical teacher that generates pseudo-data satisfying energy conservation but containing systematic biases. The RNN/LSTM [211] network is first pre-trained to minimize

$$L_{\text{pretrain}} = \frac{1}{N} \sum_i \|\hat{y}_i - \bar{y}_i\|^2,$$

obtaining a physically consistent initialization θ_{init} . The model is then fine-tuned on limited real observations, with an additional energy conservation term included in the total loss:

$$L_{\text{total}} = L_{\text{data}} + \lambda L_{\text{EC}}, \quad L_{\text{EC}} : \Delta U = F_{\text{in}} - F_{\text{out}}.$$

This two-stage learning ensures that the network starts within a physically feasible region $\mathcal{H}_{\text{phys}}$; subsequent optimization only needs to correct data bias without breaking physical consistency. The results demonstrate that the model can accurately predict lake temperature profiles while maintaining thermal balance and temporal stability even with sparse observations.

In urban systems, PINN with weight initialization has shown strong performance in multiple subdomains: *Building energy prediction*: Using EnergyPlus simulations to generate energy consumption curves for initializing network weights allows the model to start from parameter spaces consistent with heat transfer and radiation laws, shortening training time and avoiding local minima. *Traffic flow control*: In reinforcement learning for traffic signal optimization, the macroscopic flow dynamics (e.g., LWR model [212]) provide initial policy distributions, guiding the network to satisfy flow conservation automatically during learning.

This physical simulation-based parameter initialization mechanism injects physical structure into the parameter space, enabling stable, fast, and physically consistent convergence under cold-start or non-stationary conditions. It represents a typical implementation of the Physics-Integrated AI paradigm.

A.3. PINN with architecture design

This method directly embeds physical laws—such as conservation, symmetry, or topological constraints—into the network architecture itself, ensuring that the forward propagation inherently satisfies physical consistency. Unlike the “soft constraint” imposed in loss-based approaches, this is a “hard constraint” strategy, meaning the network is structurally restricted to produce only physically admissible outputs. As a result, even under perturbations (e.g., dropout or noisy data), the model maintains physically reasonable and stable predictions.

A representative example is the *Physics-Guided Architecture* (PGA) proposed by Read et al. [35]. In the lake temperature modeling task, the method enforces the physical rule that “density increases monotonically with depth” by redesigning the LSTM output structure. Instead of directly predicting temperature $\hat{T}_{t,d}$ and penalizing violations in the loss, PGA-LSTM predicts non-negative density increments

$$\Delta \rho_d = \text{ReLU}(W_d h_d + b_d),$$

and accumulates them layer by layer as

$$\rho_d = \rho_{d-1} + \Delta \rho_d.$$

This structure ensures that $\rho_d \geq \rho_{d-1}$ always holds, so the output remains physically consistent in both training and inference. A learnable mapping $T = g_\phi(\rho, x_t)$ then recovers temperature, realizing an intrinsically constrained architecture.

This design functions as a form of *structural regularization*: gradient propagation is confined to the non-negative region of ReLU, ensuring parameter updates remain within the physically feasible space. Consequently, the model gains improved interpretability and robustness. Here, physical laws define the network connectivity itself rather than acting as external penalties.

In urban systems, such “structural embedding” approaches are widely applicable: Transportation systems: GNN can encode road topology while embedding flow-conservation convolutional kernels to maintain inflow–outflow balance at every node. District energy networks: Hamiltonian Neural Networks (HNNs) can model energy exchange processes that inherently conserve total system energy during dynamic evolution.

Overall, this method exemplifies the “architecture-level constraint” pathway within the Physics-Integrated AI paradigm. By internalizing physical principles into model architecture, it ensures conservation, interpretability, and consistency under diverse data conditions—representing a critical step toward trustworthy AI modeling in complex physical systems.

A.4. Sequential physics-AI ensemble

This method adopts a sequential architecture in which physics-based and AI-based modules cooperate step by step within a unified computational pipeline. The physics component produces a baseline or coarse-grained solution, while the AI module refines it by performing residual correction, resolution enhancement, or local operator approximation. Through this process, the framework achieves accurate and efficient prediction while ensuring physical consistency. The two components jointly form a closed feedback loop of *physics solution, AI refinement, and physical projection*, enabling process-level, continuous fusion of data-driven and mechanistic knowledge.

A representative example is the turbulence modeling framework proposed by Kochkov et al. [40], in which a neural module is embedded within a CFD solver [213]. The solver iteratively advances the velocity field $u(t)$ on a coarse grid, yielding a preliminary solution $u^* = \mathcal{P}_{\text{CFD}}(u(t))$. The neural network then estimates a correction term Δu to recover high-frequency information, after which a projection step enforces physical constraints such as $\nabla \cdot u = 0$. This iterative process ensures that each state remains within the feasible physical manifold, preventing the accumulation of numerical errors. Rather than replacing the PDE, the neural network locally approximates its operators, producing a *physics backbone with data-driven correction* that combines physical reliability with learning flexibility.

This sequential design provides several notable advantages: *Physical consistency*: Each corrected solution is projected back into the space satisfying conservation and stability. *Computational efficiency*: AI approximates only local operators, reducing computational cost by orders of magnitude. *Generalization*: The preserved PDE structure allows adaptation across various Reynolds numbers and boundary conditions. *Interpretability*: The learned corrections can be directly compared with physical fluxes, yielding transparent insights into the latent dynamics.

Its practical value is evident in multiple urban contexts: *Transportation systems*: Macroscopic LWR models [212] capture global flow trends, while neural modules refine local nonlinearities caused by signals or bottlenecks, improving congestion prediction and control precision. *Urban hydrology*: Hydrodynamic solvers estimate basin-scale responses, whereas neural refinements correct rainfall-runoff biases, supporting high-resolution, real-time simulations during extreme events.

Overall, the sequential Physics-AI Ensemble represents a balanced mechanism in which physics provides a stable foundation and AI contributes adaptive refinement, offering a general pathway for large-scale coupled modeling in urban systems.

A.5. Parallel physics-AI ensemble

The parallel ensemble framework operates two models—the physics model and the AI model—simultaneously and integrates them in a high-level representation space. The physics branch captures explicit mechanisms governed by conservation equations (e.g., advection, diffusion, or energy transfer), while the AI branch learns implicit dynamics that are difficult to formalize, such as human activities, social rhythms, or unsteady disturbances. These two branches evolve concurrently in time and are aligned either in a latent space or at the output layer, forming a complementary dual-path representation of system dynamics. This bidirectional cooperation reflects the core logic of the *Physics-AI Hybrid Ensemble* paradigm.

A representative implementation is the *Air-DualODE* model [214], which couples two Neural ODEs for urban air-pollution dynamics. The physics branch is derived from an advection-diffusion equation,

$$\frac{dX}{dt} = \alpha(-kL_{\text{diff}}X) + (1 - \alpha)L_{\text{adv}}X + \beta X,$$

to model explicit transport and dilution processes, while the data branch employs an attention-enhanced Neural ODE to capture nonlinear meteorological and anthropogenic effects. The two systems are integrated synchronously and aligned through a decay-weighted temporal contrastive loss:

$$L_{\text{TCL}} = \|Z_P(t) - Z_D(t)\|^2 e^{-\lambda \Delta t},$$

which enforces consistent long-term co-evolution. A fusion layer then aggregates information via cross-attention and graph-based operations to produce final predictions.

The strengths of this parallel architecture can be summarized as follows: *Physical consistency with flexibility*: Explicit PDEs guarantee conservation, while neural operators capture complex non-physical disturbances. *Complementarity*: The physics branch provides stability, and the AI branch compensates for model misspecification in complex boundary and source-sink conditions. *Transferability*: Decoupled yet coordinated branches allow generalization across cities and climates. *Training stability*: Parallel evolution alleviates gradient conflicts and stabilizes optimization in multi-task settings.

In urban applications, this hybrid parallel framework enables integrated modeling of multi-source heterogeneous systems: *Air quality forecasting*: Physics-based chemical transport models simulate advection and diffusion, while AI branches learn emission and activity-driven fluctuations, achieving multi-scale prediction accuracy. *Urban flood simulation*: Hydrodynamic equations and spatiotemporal neural networks run in parallel to represent runoff physics and rainfall features, maintaining conservation while refining spatial detail. *Energy-transport coordination*: Physics networks encode topological constraints, and AI networks model demand-side fluctuations to support joint optimization and decision-making.

Overall, Method 5 embodies a fully bidirectional Physics-AI coupling mechanism in which physics defines theoretical boundaries and AI enhances nonlinear adaptivity—an essential configuration for maintaining physical consistency and scalability in complex, open urban systems.

A.6. AI-discovered physics model

This approach leverages sparse symbolic regression [51] and neural ODE [215] to automatically identify governing equations from observational data, thus achieving a reverse flow of knowledge from *data to physics*. The central idea is to search a library of potential physical terms and select a minimal subset that best reproduces system dynamics, thereby reconstructing the underlying control equations. This mechanism represents the essence of the *AI-Integrated Physics* paradigm, where AI contributes to physical discovery rather than mere approximation.

SINDy [51] builds a candidate library $Q(U)$ from the space-time field $u(x, t)$ (e.g., $u, u_x, u_{xx}, u^2, uu_x, u_{xxx}$) and infers u_t to identify the governing

equation:

$$u_t = Q(U) \xi,$$

where the coefficient vector ξ is sparse; nonzero components correspond to the true PDE terms. The method employs sequential thresholded ridge regression (STRidge) to stabilize ill-conditioned problems, prune weak candidates, and iteratively converge to a compact, interpretable PDE.

This framework has successfully recovered physical laws across multiple domains: *Fluid dynamics*: Reconstructing the Navier-Stokes equation [209] and estimating the Reynolds number [216] from vorticity-velocity data. *Diffusion and waves*: Discovering the heat equation and the KdV equation [217] from stochastic or solitary-wave trajectories. *Nonlinear oscillators*: Identifying key terms even under noisy or sparse sampling conditions.

In complex urban systems, such data-driven discovery offers new opportunities: *Economic modeling*: Deriving nonlinear growth equations among GDP, investment, and employment dynamics. *Traffic modeling*: Uncovering interpretable functional relationships among speed, density, and flow.

This method demonstrates three distinctive merits: *High interpretability*: It outputs explicit, symbolic equations and parameters rather than opaque weights. *Robust efficiency*: The STRidge algorithm performs reliably under noise and limited samples. *Extensibility*: It applies to both Eulerian fields and Lagrangian trajectories, supporting multi-scale and open-system dynamics.

Overall, the AI-Discovered Physics Model transforms equation discovery into a sparse-structure identification task. By inferring governing equations directly from data, it bridges the gap between empirical modeling and physical reasoning, enabling interpretable and generalizable understanding of complex urban dynamics.

A.7. Neural physics model

The Neural Physics Model preserves the explicit structure of physical equations while replacing poorly characterized or analytically intractable components with neural modules. In contrast to sequential or parallel ensembles, here the neural operator is embedded within the physical equation itself, creating an explicit-implicit hybrid mechanism that combines theoretical interpretability with data-driven adaptivity. This represents the advanced form of the *AI-Integrated Physics* paradigm.

A representative example is the *Physics-Infused Machine Learning for Crowd Simulation* model [55]. Using the Social Force Model (SFM) as the physical backbone,

$$m_i \frac{dv_i}{dt} = f_i^D + \sum_j f_{ij} + \sum_o f_{io},$$

the human-human interaction term f_{ij} is replaced by a learnable graph neural network:

$$\tilde{f}_{ij} = \phi_{\text{edge}}(s_j - s_i), \quad \sum_j \tilde{f}_{ij} = \phi_{\text{node}}\left(\sum_j \phi_{\text{edge}}(s_j - s_i)\right).$$

The neural module takes relative position, velocity, and view-angle differences as inputs, aggregates them symmetrically, and ensures consistency with Newton's third law. The result is still an explicit force equation, but with a learned rather than handcrafted interaction law.

The model is trained using a composite objective:

$$L = L_{\text{position}} + \lambda_1 L_{\text{velocity}} + \lambda_2 L_{\text{collision}},$$

where the collision term enforces physical plausibility. A ‘‘physics-teacher/data-student’’ regime pretrains the model on physically consistent SFM simulations and then fine-tunes it on real trajectories. This dual-phase training acts as internal knowledge distillation, ensuring that the learned forces remain dynamically valid while reflecting real behavioral complexity.

To validate the physical meaning of the learned operator, symbolic regression is applied to the trained network to extract an analytical formula:

$$f_{ij} = \lambda_1 e^{-d_{ij}/\lambda_2} [1 + \lambda_3 \cos(\theta_d - \theta_v) + \lambda_4 d_{ij} \cos(\theta_d - \theta_v)],$$

which retains the exponential decay from SFM but adds new angle- and distance-dependent terms, revealing nonlinear avoidance behavior. This confirms that the neural term not only fits data but also reveals new physical mechanisms.

The main advantages of this framework can be outlined as follows: *Physical fidelity*: The global conservation structure and mechanical constraints are maintained. *Adaptive learning*: Neural modules flexibly model complex, nonlinear interactions. *Interpretability*: Symbolic extraction converts learned dynamics into explicit analytical expressions. *Stability and generalization*: The hybrid form enhances stability and allows transfer to diverse scenarios.

In broader urban contexts, Neural Physics Models can be applied to wind-field reconstruction, energy-flow optimization, and crowd or traffic simulations. For instance, neural closures in Navier-Stokes [209] equations enable fine-grained urban airflow prediction, while hybridized network equations can learn local perturbations in traffic or energy networks without violating global conservation principles.

Overall, this method embeds learnable operators within explicit physical frameworks, achieving a synthesis of rigor and flexibility. It represents one of the most promising directions for developing interpretable and adaptive AI-augmented physics models in complex urban systems.

Appendix B. Comparative analysis with conventional AI

A series of recent representative studies illustrate how PIAI can systematically enhance the robustness, interpretability, and generalization of conventional data-driven models in urban systems. For instance, Yu et al. [218] proposed a graph-based reinforcement learning framework for coordinated multi-intersection traffic signal control. Within the PIAI paradigm, this task can be advanced by embedding physical constraints—such as vehicle conservation, phase safety, and capacity limits—directly into the optimization objective through the *PINN with loss function* strategy. This integration ensures consistency with traffic-flow physics, stabilizes policy learning under perturbations, and provides interpretable indicators such as conservation residuals and conflict rates.

Similarly, Yu et al. [219] developed a robust multivariate forecasting model addressing missing and asynchronous urban sensing data. When extended by the *Parallel Physics-AI Ensemble*, a dual-branch design can be implemented, where a physical branch models conservation or diffusion dynamics while an AI branch captures nonlinear residuals. Their latent-space fusion maintains physically coherent forecasts under high missingness, suppresses drift, and improves transferability across cities, realizing joint robustness and interpretability.

In [220], a Transformer-based framework was introduced for efficient large-scale traffic forecasting. Under the PIAI framework, this approach can incorporate a *Sequential Physics-AI Ensemble* by introducing a macro-level physical backbone (e.g., LWR or CTM) to capture global traffic-flow continuity, combined with a neural refinement layer for local residual corrections. The resulting hybrid model preserves computational efficiency while enforcing conservation and long-term stability, providing a transferable and auditable foundation for large-scale traffic prediction.

Further, Xu et al. [221] proposed a continuous-time spatio-temporal modeling framework based on Neural ODEs. Through the same sequential Physics-AI integration, the macro component encodes system-level conservation and stability, while the micro branch learns short-term deviations under physical regularization. This mechanism introduces explicit physical semantics, enhances numerical stability, and ensures interpretable long-term predictions, demonstrating PIAI's potential in hierarchical dynamical modeling.

In the reinforcement learning domain, Mi et al. [222] presented a dynamic ensemble policy optimization model for adaptive traffic-speed control. The PIAI perspective extends it via the *PINN with weight initialization* approach, embedding flow-conservation priors and physics-regularized rewards during training. This ensures safe exploration, faster convergence, and physically interpretable state representations, thus coupling data-driven optimization with physics-consistent constraints for more reliable and explainable urban control.

Finally, Yu et al. [223] addressed multivariate forecasting under dynamically changing missingness through a teacher-student distillation framework. Enhanced by the *Sequential Physics-AI Ensemble*, the teacher network serves as a physics-informed generator minimizing both data loss and physical residuals, while the student model refines predictions under physics-consistency regularization. This integration achieves stability and interpretability even with incomplete observations, enabling a transition from statistical to physical robustness and underscoring PIAI's capability to strengthen resilience in urban sensing and forecasting.

Appendix C. Applied significance of PIAI in urban systems

This appendix provides an extended discussion on the practical impact of PIAI in real-world urban infrastructure management. Building upon Section 3 of the main manuscript, it summarizes the operational benefits, performance improvements, and cross-domain applications of PIAI across eight representative urban subsystems. Two complementary perspectives are presented below: (I) the general advantages of PIAI over conventional AI methods; and (II) the specific performance improvements achieved in major urban domains.

C.1. Advantages of PIAI over conventional AI

Unlike traditional data-driven AI models, PIAI explicitly embeds physical laws—such as conservation equations, continuity constraints, and dynamical invariants—into its model structure or optimization objective. This integration ensures *physical consistency*, *data efficiency*, and *transferability*, enabling PIAI models to remain stable, interpretable, and trustworthy under complex, dynamic, and safety-critical urban environments. To make these advantages more concrete, we summarize below six key dimensions where PIAI provides quantifiable performance gains:

Accuracy and Timeliness. By incorporating physical constraints, PIAI reduces model bias and improves generalization. Sequential and parallel hybrid architectures further enable fast inference and high-frequency updates, allowing dynamic visualization and real-time response in city operations.

Computational Efficiency. Neural networks can approximate costly physical solvers (e.g., in fluid, hydrological, or energy systems), learning the operator's latent solution space while preserving the underlying physical structure. This substantially reduces computational complexity and supports near real-time optimization and scheduling.

Operational Optimization. By embedding physics into both prediction and optimization stages, PIAI enables decision-making based on explicit physical laws—such as energy conservation, power-flow balance, or traffic continuity—rather than empirical heuristics. In energy or transportation dispatching, the model outputs can be directly integrated into control equations or optimization constraints, ensuring that predicted values remain within the physically feasible domain and that resulting decisions are interpretable and verifiable.

System Reliability. Built-in physical consistency mechanisms prevent overfitting and non-physical model collapse, ensuring stable and valid behavior even under extreme operating conditions (e.g., heatwaves, heavy rainfall, or peak loads).

Resilience and Risk Reduction. Because PIAI explicitly models disturbance propagation and spatiotemporal dynamics (e.g., through advection-diffusion equations or coupled network dynamics), it can

simulate disaster impacts and recovery trajectories with interpretable physical fidelity. In risk scenarios, physically consistent models generate reliable response mappings, helping to identify vulnerable components and critical nodes, while supporting physically grounded resource reallocation and post-event recovery optimization.

Decision Support. Sequential and parallel PIAI architectures can jointly learn the coupling among multiple urban subsystems within a unified physical constraint framework—for instance, the interplay between energy conservation and flow continuity across energy, transportation, and environmental networks. Such integrated structures allow models from different domains to share physical variables (e.g., flow, energy, or pressure fields), ensuring conservation and feedback consistency in multi-domain digital twins, and enabling coordinated simulation and joint decision support across systems.

These additions make the discussion more application-oriented and demonstrate how PIAI not only improves model performance but also provides *operationally measurable benefits* in real urban infrastructure management.

C.2. Practical achievements and comprehensive improvements across eight urban subsystems

In the revised manuscript, we further expand the discussion by incorporating eight representative urban subsystems and illustrating their performance gains across the six key improvement dimensions—namely, *accuracy and timeliness*, *computational efficiency*, *operational optimization*, *system reliability*, *risk response and resilience*, and *cross-system decision support*. The following sections summarize the actual mechanisms through which PIAI enhances each subsystem.

Energy: Intelligent Energy Dispatch and System Stability Assurance. Key improvements: accuracy, efficiency, operational optimization, reliability.

PIAI integrates neural network learning with power-flow equations and energy conservation constraints to enable high-precision forecasting of power demand and energy flow, thereby reducing the dependency on high-frequency measurement data and improving both accuracy and timeliness. By approximating complex power-flow solvers with neural operators, the model significantly reduces simulation complexity and enables efficient energy distribution and operational emulation. Based on physically consistent outputs, adaptive scheduling among generation, storage, and loads becomes feasible, minimizing manual intervention and scheduling latency. Moreover, the embedded physical constraints ensure voltage and frequency stability under abnormal fluctuations or extreme load conditions, greatly enhancing the system's reliability and steady-state control.

Environment: Air Quality and Atmospheric Dynamics Forecasting. Key improvements: accuracy, efficiency, risk response.

PIAI combines advection-diffusion equations with neural ordinary differential equation frameworks, allowing continuous pollutant field reconstruction from sparse observations and improving spatial prediction accuracy and temporal responsiveness. In terms of efficiency, neural approximations replace part of the high-dimensional transport equation's numerical integration, reducing iterative computation while enabling rapid simulation of pollutant dispersion and meteorological evolution. Meanwhile, physics-guided constraints allow the model to capture wind-driven pollutant propagation patterns, enhancing risk responsiveness to extreme weather and pollution events and providing physically interpretable support for early warning and intervention.

Economy: Economic Dynamics and Resource Allocation Optimization. Key improvements: operational optimization, reliability, decision support.

Using symbolic regression and sparse dynamics frameworks, PIAI automatically discovers underlying economic growth laws and structural constraints from macroeconomic data, resulting in interpretable

and physically consistent predictions. By incorporating conservation and feedback mechanisms into macroeconomic models, PIAI identifies critical pathways of resource flows, supporting policy-level optimization and operational scheduling. The built-in physical consistency suppresses random fluctuations inherent in purely statistical models, enhancing reliability and stability under multi-factor disturbances and providing scientifically grounded insights for fiscal and industrial decision-making.

Transportation: Traffic Flow Forecasting and Signal Control Optimization. Key improvements: accuracy, efficiency, operational optimization, reliability.

By embedding fluid-dynamic equations (e.g., the LWR model) into deep learning frameworks, PIAI enforces flow continuity and vehicle conservation constraints, improving the spatiotemporal accuracy of traffic predictions. The sequential Physics-AI ensemble structure uses low-resolution physical simulations as inputs while AI modules refine local patterns and correct residuals, thus improving computational and simulation efficiency. The physically consistent model outputs can be directly integrated into signal optimization equations, enabling dynamic control and intelligent path assignment while reducing delays and energy consumption. Moreover, physical consistency ensures model stability under extreme traffic conditions, enabling reliable management of high-density mobility scenarios.

Information: Communication and Resource Scheduling. Key improvements: efficiency, operational optimization, reliability, decision support.

PIAI embeds flow-conservation and interference-propagation laws into graph neural network models, enhancing the physical rationality of bandwidth allocation and load forecasting. The model learns time-varying transmission properties across complex topologies, enabling rapid identification of bottleneck nodes and latency distributions and improving overall computational and scheduling efficiency. Through physically consistent node-interaction modeling, the system achieves autonomous load balancing and link adaptation, maintaining reliable transmission even under network fluctuations or node failures and providing interpretable decision support for network optimization.

Public Services: Healthcare, Education, and Infrastructure Scheduling. Key improvements: operational optimization, reliability, decision support.

PIAI incorporates facility capacity, population distribution, and spatiotemporal mobility patterns into a physically constrained framework, enabling demand forecasting that dynamically aligns with service supply. This mechanism optimizes cross-regional allocation and utilization of healthcare, education, and infrastructure resources, ensuring that operational decisions remain interpretable and verifiable. The model also dynamically evaluates the stability of service networks, maintaining resilience and recoverability under sudden demand surges or resource imbalance, thereby supporting efficient and equitable public service management.

Emergency Management: Disaster Response and Urban Resilience Enhancement. Key improvements: accuracy, risk response, decision support.

PIAI models disaster propagation, energy disruption, and social response behaviors as coupled dynamical systems, using neural networks to learn unknown disturbance terms for multi-hazard scenario simulation. Physical constraints preserve conservation of energy, mass, and information during simulation, strengthening scientific rigor in emergency scheduling and resource deployment. The framework autonomously generates feasible response pathways and priority sequences for different disturbance types, providing decision support and dynamic simulation capabilities for urban risk management and resilience planning.

Urban System as a Whole: Multi-Domain Coordination and Digital Twin Support. Key improvements: efficiency, reliability, decision support.

At the city-wide level, PIAI enables unified modeling across energy, transportation, environmental, and economic systems through a multi-domain integration framework. Its parallel mechanisms that share conserved variables and constraints ensure consistent cross-system feedback

and stable model convergence, facilitating synchronized simulation and policy alignment. This structure grounds digital twin models in physically interpretable foundations while maintaining high computational efficiency, providing system-level decision support for policy formulation and multi-scenario evaluation.

In summary, the application of PIAI across these eight subsystems demonstrates a systematic evolution from single-domain performance enhancement to cross-domain, decision-oriented urban governance. By integrating physical consistency to improve credibility and data-driven learning to enhance flexibility and efficiency, PIAI enables a paradigm shift in urban infrastructure management from empirical optimization toward physically verifiable and intelligently coordinated decision-making.

Appendix D. Future research directions for PIAI

This appendix elaborates the research pathways that turn the open problems in Section 5.5 into an actionable agenda for scalable, generalizable, and verifiable PIAI in urban systems.

D.1. Theoretical foundations: Convergence, generalization, and error bound

Convergence PIAI models jointly minimize data-driven loss and physics-based constraints, leading to highly non-convex optimization landscapes prone to oscillation or local minima. This challenge is exacerbated in urban systems where multi-scale and multi-process couplings are common. We propose improving convergence from two complementary perspectives: *Structured convergence design*: by embedding conservation laws and symmetries into the network architecture, parameter updates remain within a feasible physical manifold, ensuring convergence to physically consistent equilibria. *Layered and decoupled optimization*: decomposing urban systems into micro-meso-macro levels and enforcing conservation at the interfaces reduces gradient oscillation across scales, ensuring stable and interpretable training dynamics.

Through these design principles, PIAI achieves controllable, interpretable, and reproducible convergence pathways suitable for engineering-scale urban simulations.

Generalization Most current PIAI models perform well in single cities but degrade significantly under cross-city or cross-domain transfer. Deep learning models typically learn local statistical correlations, whereas urban generalization depends on the universality of physical laws. Generalization can be strengthened through two complementary routes: *AI mechanisms*: transfer learning and meta-learning enhance cross-domain adaptation by reusing shared parameters and learning-to-learn strategies for few-shot transfer. *Physical mechanisms*: universal law discovery through neural-symbolic reasoning, equation discovery, and operator learning extracts invariant relations from multiple urban datasets to serve as transferable physical priors.

The combination of adaptive AI and invariant physics forms a structured generalization pathway, enabling PIAI to transfer knowledge seamlessly across energy, transportation, and environmental systems.

Error Bound Current PIAI studies often rely on empirical metrics without theoretical upper or lower bounds, making it difficult to quantify the reliability and trust region of predictions under dynamic conditions. We suggest defining theoretical error bounds from two complementary perspectives: *Information-theoretic lower bound*: using predictability analysis and the Bayes Error Rate (BER) or Mutual Information Lower Bound to estimate the minimal achievable error [224]. *Dynamical upper bound*: using Lyapunov spectra or perturbation propagation models to characterize how errors amplify over time and space, thereby identifying stability limits in chaotic regions.

Together, these analyses define a “predictable interval” within which PIAI performance remains optimal and risk-controlled, marking a shift from zero-error pursuit to uncertainty-aware reliability.

D.2. Unified modeling of heterogeneous physical laws: Cross-scale integration challenges

Problem and Challenge Urban systems inherently involve multiple modeling paradigms across scales: the macro level typically uses deterministic partial differential equations (PDEs) to represent continuous conservation processes (e.g., energy flow, traffic density); the meso level employs stochastic differential equations (SDEs) to describe uncertainty propagation (e.g., emission diffusion, load fluctuation); and the micro level relies on agent-based models (ABMs) to capture individual decision-making and behavioral interaction (e.g., mobility, device control). These paradigms differ fundamentally in mathematical assumptions and representational forms—PDEs assume differentiability, SDEs capture stochastic evolution, and ABMs are event-driven and discrete-making their integration a core challenge rather than merely a domain-transfer problem.

Proposed Solution AI can serve as a “mediating language” across modeling paradigms by bridging continuous, stochastic, and discrete formulations through three mechanisms: *Representation Unification*: deep models embed PDE, SDE, and ABM behaviors in a shared latent space through probabilistic density or conservation representations, providing mutual translatability among modeling forms. *Cross-Scale Mapping*: AI learns implicit operator relationships between modeling levels—e.g., inferring macroscopic PDEs from microscopic ABM behaviors, or deducing agent strategies from macroscopic flow distributions—reducing human-defined averaging assumptions and cumulative errors. *Differentiable Coupling*: neural operators and probabilistic graph networks integrate outputs of PDEs, SDEs, and ABMs into a single differentiable computation graph, enabling end-to-end optimization and feedback consistency across scales.

This integration pathway is theoretically grounded: the cross-level mapping learned by AI can be interpreted as a *learnable mean-field operator*, aligning with emergence principles in multi-scale systems. From an information-theoretic perspective, the latent-space embedding acts as a minimum description length compression of high-dimensional discrete states, achieving efficient multi-scale representation. Urban systems—composed of individual decisions, stochastic perturbations, and continuous evolution—naturally fit this unified modeling framework. For example, in coupled transportation–energy–environment systems, individual driving and consumption behaviors (ABM) influence macro-level flow and diffusion processes (PDE) through stochastic disturbances (SDE), while PIAI captures these causal chains within a consistent cross-scale structure.

D.3. Noise, missingness, and bias: Robustness under real-world data conditions

Problem and Challenge While PIAI models achieve consistent physics-data learning under ideal data, real urban datasets typically contain measurement noise, missing observations, and sampling bias. These issues disrupt the correspondence between physical constraints and observed signals, causing gradient oscillation, constraint mismatch, and cross-domain performance degradation, thereby undermining stability and trustworthiness. Strengthening robustness under imperfect data is therefore essential for engineering-grade deployment at city scale. Noise distorts the computation of physics residuals; missingness breaks spatiotemporal continuity and conservation; bias drives overfitting to local regularities while neglecting invariant physics. Classic data-driven fixes (e.g., interpolation, resampling) help partially but lack physics consistency and cannot guarantee energy, flow, or mass conservation.

Proposed Solution We introduce a physics-driven robustness framework along four complementary axes: *Physics-guided regularization*. Embed conservation laws, boundary conditions, and stability constraints in the loss so that, even under noise, optimization is “pulled back” to the feasible physical manifold; this adds structural priors to the hypothesis space and suppresses the influence of outliers. *AI-discovered physics*.

Use symbolic regression, neural operators, or dynamics identification to distill stable evolution laws from multi-source observations and inject them as hard or soft constraints during pretraining or model design; this extracts invariant structure from noisy data and strengthens transferability. *Reconstruction and self-supervision*. Reconstruct missing or corrupted inputs via self-supervised or generative models and filter the reconstructions using physics constraints, yielding physics-consistent preprocessing before the main task and improving downstream robustness at its source. *Similar-data initialization*. Initialize with pretrained weights from similar cities, time periods, or shared physical regimes so parameters start near the physical manifold, reducing unstable convergence, overfitting, and gradient jitter in “dirty” data settings.

In combination, these mechanisms enable PIAI to maintain conservation, stability, and interpretability under noise, missingness, and bias—a multi-layer “physics-first” defense that has shown tangible robustness gains in air quality, traffic flow, and energy load applications.

D.4. Cross-domain generalization and transferability: Beyond city-specific models

Problem and Challenge Many PIAI models are trained in a single city or scenario. Heterogeneity in climate, terrain, infrastructure, social behavior, and sampling protocols limits cross-city generalization; performance often drops markedly when transferring to new environments. Achieving robust cross-domain transfer is pivotal for “pan-urban intelligence” and city-scale digital twins.

Proposed Solution We organize six complementary strategies spanning physical structure, data distribution, initialization, and model design: *Universal law discovery*. Use symbolic regression, neural operators, or equation discovery over multi-city datasets to extract structural invariants (conservation, symmetries, coupling forms) and codify them as shared constraint layers or equation templates; migration then requires only boundary or parameter adjustment while preserving interpretability. *Domain adaptation*. Reduce inter-city distribution gaps in latent space via adversarial alignment, multi-task learning, or subspace projection while retaining physics consistency penalties to prevent violation of conservation during alignment. *Similar-city initialization*. Quantify inter-city similarity (climate, topography, infrastructure, socioeconomics) and transfer pretrained weights or priors among similar cities for faster convergence, fewer samples, and sustained physical consistency. *Meta-learning and few-shot fine-tuning*. Train meta-parameters across multiple cities and tasks to learn-to-learn rules that enable rapid adaptation to new locales with limited samples. *Learning to generalize with large models*. Leverage the representation capacity of large models trained on diverse tasks and physical processes to internalize cross-domain physical regularities and support structural adaptation. *Physics-informed foundation models*. Move toward unified models spanning cities, physics, and socio-data so that energy, transport, environment, and information subsystems share a transferable structural backbone for rapid reuse and multi-domain digital twins.

Together, these routes form a coherent path from local models to transferable frameworks, enabling PIAI to remain physically consistent and stable across heterogeneous urban environments and supporting large-scale reuse and deployment.

D.5. Limits in symbolic reasoning and causal modeling: A gap in interpretability

Problem and Challenge Despite gains in accuracy and generalization, many PIAI models remain black boxes, offering limited insight into causal mechanisms and physical logic—precisely what scientific computing and urban governance require. Decision-makers care about “why” and “how to control,” not just predictions. Current practice often relies on implicit deep features without symbolic structure or causal direction, hindering actionable explanations and interventions.

Proposed Solution We propose a three-layered physics-causal reasoning stack: *Symbolic equation discovery*. Extract explicit symbolic laws from latent representations via symbolic regression, sparsity, or neural-symbolic hybrids, converting learned relations into verifiable analytic structures and building reusable physics knowledge bases (e.g., key drivers of traffic flow changes, nonlinear load responses to weather). *Causal structural modeling*. Embed structural causal models or graph-based causal networks within PIAI, combined with intervention learning and counterfactuals, to distinguish correlation from causation and guide controllable variables and policy levers (e.g., emission-dispersion-health risk chains; price-demand-stability links). *Physics-aware explainable embeddings*. Constrain internal attention and representations to map back to physical quantities, fluxes, or node/edge contributions, yielding "physical attribution maps" that trace factor contributions to outputs; physics-based attention and explainable GNNs quantify influence in dynamic networks. *Neural-symbolic integration*. Combine continuous neural estimation with discrete logical reasoning (neural logic programming, knowledge-graph-augmented networks, differentiable logic constraints) to close the loop from perception to reasoning. In practice (e.g., emergency response or traffic control), such models can surface explicit rules ("congestion rises if flow exceeds threshold and signals remain unadjusted"), enabling structured urban reasoning.

By elevating symbolic and causal structure, PIAI progresses from function approximation to structure discovery, ensuring verifiable, intervenable, and transferable models that support scientifically grounded decisions across transportation, pollution control, and energy operations.

D.6. Deployment bottlenecks: Scale, cost, and dynamic validation

Problem and Challenge Although PIAI performs strongly in research settings, city-scale deployment is constrained by system size, compute cost, and the need for continual validation. High-dimensional couplings, streaming data, and resource limits complicate the simultaneous pursuit of accuracy, efficiency, and trust. A deployable PIAI stack must therefore be scalable, economical, and dynamically verifiable.

Proposed Solution We outline four system-level pillars: *Modular and hierarchical computation*. Partition large urban systems into locally learnable, globally coordinated layers: local PIAI models operate on sub-regions (districts, subnetworks), while a higher-level coordinator enforces shared boundary conditions and coupling constraints. With distributed computing and GNN-based partitioning, this supports parallel inference and "local real time-global synchronization" while preserving physics. *Lightweight and cost-efficient modeling*. Apply knowledge distillation, pruning, neural operator compression, and mixed-precision inference to reduce size and latency without sacrificing physics consistency, enabling real-time deployment on controllers, edge nodes, and IoT devices. *Dynamic data assimilation and online adaptation*. Continuously ingest streams via Kalman filtering, incremental training, or sliding-window optimization, with physics constraints to resist short-term noise overfitting and to maintain stability under shocks (pollution spikes, congestion waves, load anomalies). *Automated validation and trust monitoring*. Establish city-scale monitoring that tracks prediction error and physics residuals in operation, coupling uncertainty quantification (confidence intervals, risk boundaries) with visualization and real-time anomaly detection for self-diagnosis and self-correction over long horizons.

Taken together, these pillars chart a path from research prototypes to operational systems-delivering efficient, low-cost, and continuously verified PIAI deployments that underpin reliable, evolvable, and trustworthy urban digital twins.

Appendix E. Data availability and mitigation strategies

This appendix provides an extended discussion of the data ecosystem underlying PIAI research in urban systems. It summarizes represen-

tative datasets across major urban subsystems, highlighting their structural characteristics, application relevance, and accessibility. Furthermore, it elaborates on the principal challenges-such as sparsity, heterogeneity, privacy, and temporal evolution-that constrain large-scale deployment of PIAI, together with corresponding mitigation strategies including physics-consistent data generation, multimodal fusion, federated learning, and cross-timescale modeling. The following sections present detailed examples and current progress in each aspect.

E.1. Data foundations of urban systems

The data ecosystem of urban systems can be organized into eight major subsystems-energy, environment, economy, transportation, information, public services, emergency management, and the integrated urban system. Each subsystem has distinct data types, sources, and representative use cases:

Energy. Energy data cover grid topology, transmission and distribution networks, and end-use energy consumption. Representative public datasets include *PyPSA-Eur* (European power transmission system) [225], *PowerGraph* (benchmark power network topologies) [226], and multi-city electricity load monitoring datasets [227]. These resources support load forecasting, power flow computation, and distributed energy scheduling, forming the foundation for PIAI research on energy stability and control.

Environment. Environmental data include meteorological observations, pollutant emissions, air and water quality monitoring, and satellite remote-sensing products [228,229]. Representative datasets comprise ground-based monitoring records, satellite-derived imagery, and joint air-meteorology time-series streams. Such multi-source spatiotemporal data provide physical constraints that enable PIAI models to perform diffusion-based pollutant tracing and air quality prediction under fluid-dynamics guidance.

Economy. Economic data encompass business registration, industrial input-output tables, consumption statistics, and employment structures [230]. Recently developed spatial economic datasets (e.g., *Open Data Economy Map*) embed economic activities into geographic space, supporting PIAI studies on urban resilience, growth dynamics, and policy simulation [231].

Transportation. Transportation is among the richest urban data domains. Typical resources include multi-city road traffic datasets (e.g., Beijing, LA, Bay Area), human mobility trajectories and contact networks (*Urban Mobility* series), and Taxi/Metro OD datasets [232,233]. These datasets enable PIAI models to advance research on traffic prediction, signal optimization, and route scheduling.

Information. Information data include communication topology, temporal network flow, and interaction logs. Representative datasets such as *GenNet* [234] (a synthetic data ecosystem for mobile networks) simulate dynamic network loads, providing a testbed for studying information diffusion, communication resilience, and traffic optimization within the PIAI framework.

Public Services. This subsystem focuses on education, healthcare, and infrastructure services. Several cities have released open data on school distribution, hospital bed occupancy, facility accessibility, and service utilization [235,236]. These datasets support PIAI-based analyses of spatial service equity and resource allocation optimization.

Emergency Management. Data in this subsystem include disaster response, evacuation routes, emergency logistics, and event propagation. Representative resources include disaster drill trajectories, population mobility, and epidemic spread networks-enabling PIAI models to simulate risk propagation and optimize emergency response strategies [237,238].

Urban System as a Whole. Recently, several multimodal urban datasets have begun to integrate heterogeneous data sources such as transportation, energy consumption, and environmental monitoring [239–241]. These datasets represent an emerging trend toward cross-

domain data fusion, providing partial but valuable support for PIAI's system-level learning and multi-domain modeling.

Overall, these eight subsystems form a multi-layered data ecosystem spanning infrastructure, social behavior, static statistics, and dynamic streaming data. Such diverse and multi-scale data constitute the empirical foundation for PIAI-based prediction, generation, simulation, and decision-making. However, their heterogeneity, uneven coverage, and asynchronous availability also pose substantial challenges-addressed in the next subsection.

E.2. Data challenges and mitigation strategies

Despite the large amount of data accumulated across the subsystems, significant gaps in *availability and usability* remain. The main issues arise in four dimensions: sparse and missing observations, system heterogeneity and coupling difficulty, privacy and data access constraints, and temporal drift due to long-term evolution. Recent PIAI research has developed complementary solutions spanning synthetic data generation, multi-source fusion, federated collaboration, and cross-timescale modeling to address these issues.

Sparse and incomplete observations. Sensor deployment is uneven, sampling is discontinuous, and failures cause gaps in time and space. This can be mitigated by combining physics-based simulation with AI-driven generative augmentation. Simulators governed by conservation or dynamical equations can generate virtual samples (e.g., for energy, pollution, or traffic systems), while GANs, diffusion models, and neural ODEs synthesize physics-consistent data. Neural operators and spatiotemporal Transformers can reconstruct missing observations under physical constraints, yielding continuous dynamics without extra sensors.

Heterogeneity and multimodal coupling. Urban data span traffic, energy, environment, and human behavior with varying formats, scales, and accuracies. Physics-constrained data fusion mechanisms address this issue: GNNs and multimodal Transformers align cross-modal representations in latent space, while conservation or topological balance constraints ensure physically consistent fusion. Knowledge graphs further link subsystems semantically, enabling coherent multi-domain representation for digital twins.

Privacy and data access constraints. Energy, communication, and emergency data often involve sensitive or confidential information. Federated and collaborative learning frameworks allow distributed PIAI training: local models learn independently and share only parameters or gradients. The central node aggregates updates under physics consistency constraints, ensuring coordination without exposing raw data-thus enabling privacy-preserving knowledge transfer and cross-city collaboration.

Temporal evolution and drift. Urban systems evolve across multiple time scales, where short-term perturbations and long-term trends coexist. Cross-timescale physical modeling integrates short-term dynamics and long-term physical constraints within a single model. This allows capturing high-frequency events (e.g., congestion, weather anomalies) while maintaining conservation and steady-state coherence over longer horizons, enhancing predictive stability and reliability.

In summary, data challenges in urban systems-sparsity, heterogeneity, privacy, and dynamics-are being addressed within a unified PIAI framework. Through synthetic data generation, multi-source fusion, federated learning, and cross-timescale modeling, PIAI achieves physically consistent, robust, and transferable performance even under incomplete or constrained data, laying a solid foundation for cross-domain urban intelligence.

Appendix F. Standardized Evaluation metrics and benchmarking

This appendix outlines the evaluation framework and benchmarking criteria for assessing Physics-Informed AI models in urban contexts.

It complements the methodological discussions by defining standardized metrics that jointly capture predictive performance, physical consistency, interpretability, robustness, and systemic impact. The framework also identifies representative benchmark datasets and evaluation protocols across energy, transportation, environment, and multimodal urban domains, aiming to promote comparability and reproducibility of PIAI studies. Detailed metric definitions and dataset mappings are provided in the following subsections.

F.1. Predictive and data-level metrics

These metrics assess the model's accuracy, stability, and responsiveness in temporal and spatial prediction tasks: RMSE / MAE / R^2 / NSE: evaluate predictive errors and goodness-of-fit. Data coverage and sampling frequency: quantify the completeness of multimodal urban datasets (e.g., remote sensing, video streams, IoT sensors, API feeds). Latency and responsiveness: measure real-time inference efficiency under streaming data conditions.

These indicators correspond directly to the "data modality-collection-update frequency" triad in modern urban sensing infrastructures, ensuring alignment with the operational realities of heterogeneous and time-sensitive data ecosystems.

F.2. Physical consistency metrics

Since a primary objective of PIAI is to ensure that AI models learn and adhere to physical laws, explicit physics-based metrics are introduced: Conservation Error (CE): deviation in energy, mass, or flow conservation. PDE Residual or Physics Residual (PR): $\|f_{\theta}(x, t) - \mathcal{F}(x, t)\|$, measuring the residual between model outputs and governing equations. Boundary Condition Deviation (BCD): degree of compliance with physical boundary constraints. Stability Index or Lyapunov Margin (SI/LM): evaluates numerical stability and convergence under iterative inference.

These metrics ensure that learning outcomes remain within the feasible physical manifold, aligning with the "key factors-indicator relations" framework in the revised manuscript, and linking physical principles directly to evaluative criteria.

F.3. Interpretability and causality metrics

These metrics measure the transparency and scientific interpretability of model outputs: Symbolic Fidelity Score (SFS): consistency between AI-discovered symbolic equations and true physical laws. Causal Alignment Ratio (CAR): whether learned dependencies align with causal directions commonly observed in urban dynamics, such as the relationship among land use, traffic, emissions, and public health. Indicator Traceability (IT): the ability to map AI outputs back to interpretable factor-indicator hierarchies for policy-level explanation.

This dimension aligns with the "functional structure-urban metabolism-indicator system" logic discussed in the reference framework, emphasizing multi-level interpretability from macro factors to quantitative indicators.

F.4. Cross-domain generalization and robustness metrics

These metrics quantify a model's transferability and adaptability across cities, environments, or imperfect data conditions: Domain Generalization Gap (DGG): performance drop between source and target domains. Noise Robustness Ratio (NRR): performance retention under varying noise levels. Missing-Rate Stability (MRS): error stability as a function of data missingness. Adaptation Efficiency (AE): required sample size and convergence time for cross-city adaptation.

These metrics are particularly relevant for multimodal urban data-combining remote sensing, video, text, and structured sources-where heterogeneity and asynchronous sampling impose strong robustness requirements.

F.5. System-level and policy-relevant metrics

This dimension evaluates the systemic and decision-support capabilities of PIAI models: Functional State Consistency (FSC): whether predictions preserve dynamic balance among energy, transportation, environment, and social subsystems. Indicator Coupling Score (ICS): accuracy in capturing dynamic interdependencies between key urban indicators. Urban Sustainability Index (USI): composite metric integrating energy efficiency, emission control, and service accessibility.

These system-level metrics align with the “urban complexity and key factor quantification” framework, highlighting PIAI’s role in enabling interpretable and policy-relevant decision support for sustainable urban governance.

F.6. Benchmark datasets and evaluation protocols

Representative datasets are adopted to ensure standardized evaluation across PIAI methods. Energy: PyPSA-Eur [225], PowerGraph [226]; Environment: WRF-Chem, OpenAQ [228,229]; Transportation: METRLA, PEMS-BAY, CityFlow [232,233]; Information and Communication: GenNet [234]; Multimodal and Integrated Systems: CityLearn, UUKG [239,241]. Evaluation follows standard protocols such as chronological splits, cross-city transfer, and ablation on physics-informed terms, with metrics covering prediction (MAE, RMSE, MAPE), robustness (variance across runs), and physical consistency (PDE residuals, conservation errors). Together, these datasets and protocols provide a unified and reproducible basis for benchmarking Physics-Integrated, Hybrid-Ensemble, and AI-Integrated Physics paradigms.

Together, these metrics and benchmark datasets establish a comprehensive evaluation framework for PIAI models in urban systems, enabling standardized comparison across methods while ensuring alignment with physical consistency, interpretability, and decision relevance.

Appendix G. Comparative overview of PIAI methods

This appendix expands the methodological overview in the main manuscript by organizing the seven representative PIAI approaches into a unified narrative and a comparative table. The goal is to facilitate rapid selection and comparison for practical applications while clarifying principles, advantages, limitations, and typical urban scenarios.

A detailed comparison of the seven representative PIAI methods is provided below to complement the tabular summary. Each method reflects a distinct depth of physics-AI integration, with unique advantages and trade-offs in interpretability, robustness, and computational complexity.

Method 1 Advantages: Highly generalizable and compatible with any deep learning architecture (MLP, GNN, Transformer, etc[242]). By embedding explicit residual or conservation terms into the loss, this approach enforces physical consistency and improves robustness under sparse or noisy data conditions. It is particularly suitable for continuous-field modeling constrained by multi-physics equations or complex boundary conditions. **Limitations:** The choice of loss weighting (λ) strongly affects training stability; gradient-scale mismatch between physical and data terms may cause optimization competition; and conservation constraints often weaken during inference when the loss is no longer active.

Method 2 Advantages: Initializing parameters within physically meaningful ranges accelerates convergence, reduces data requirements, and stabilizes early training. This approach is especially effective in small-sample, cold-start, or simulation-assisted learning settings, mitigating overfitting and numerical instability. **Limitations:** Strongly dependent on the quality of prior physical models-systematic bias in the initial solution may lead to suboptimal convergence. If no physical regularization is maintained during training, the model may drift away from the feasible

manifold over time. Applicable primarily to systems with computable initial approximations.

Method 3 Advantages: Guarantees conservation or symmetry at the architectural level (e.g., Hamiltonian, Lagrangian, or group-invariant structures), ensuring physical consistency by design. Provides strong inductive bias and cross-domain generalization, with interpretable correspondence between network modules and physical operators. **Limitations:** Requires system-specific customization, reducing universality; rigid structural constraints may limit flexibility under abnormal or perturbed conditions; and implementation often depends on differentiable physical solvers or automatic differentiation support.

Method 4 Advantages: Balances physical consistency with AI flexibility—the physics module provides a stable baseline, while the AI component refines residuals and nonlinear deviations. Achieves substantial computational acceleration without sacrificing accuracy, offering controllable error and strong numerical stability. **Limitations:** Coupling between physical solvers and neural networks can be complex; non-differentiable solvers hinder end-to-end training; and if the AI correction exceeds the feasible physical domain, multi-step prediction errors may amplify.

Method 5 Advantages: Simultaneously models explicit physical processes and implicit dynamics, capturing cross-boundary or externally driven effects in open systems. Provides a balance between robustness and flexibility, dynamically adjusting between physical reliability and data adaptiveness. **Limitations:** Difficult alignment of output scale and phase between branches; complex weighting and fusion; and increased computational and memory costs during parallel inference.

Method 6 Advantages: Automatically identifies governing equations and key parameters, uncovering underlying physical laws rather than performing pure numerical fitting. Highly interpretable and theoretically insightful, capable of revealing hidden couplings and unknown mechanisms. **Limitations:** Requires high data quality, dense sampling, and accurate boundary conditions; omission of key terms from the candidate function library hinders recovery of the true equations; limited adaptability to discrete or highly stochastic systems.

Method 7 Advantages: Maintains the explicit equation structure while locally completing unknown mechanisms through learnable neural operators. Combines physical consistency with adaptivity, inheriting conservation and convergence properties of the original PDE. Outputs are stable, interpretable, and easily deployable; neural modules can be replaced or symbolically regressed for mechanism reconstruction. **Limitations:** Complex model design requiring manual specification of embedded variables, operator placement, and gradient paths; high computational cost due to frequent PDE solving and backpropagation; best suited for well-structured systems with partially unknown mechanisms.

Overall, Methods 1–3 correspond to the *Physics-Integrated AI* paradigm, focusing on embedding physical knowledge during training or within architecture-straightforward to implement but with limited long-term conservation. Methods 4–5 belong to the *Physics-AI Hybrid Ensemble* paradigm, integrating physical laws and data-driven models at the system and process levels, achieving a balance between interpretability and generalization. Finally, Methods 6–7 represent the *AI-Integrated Physics* paradigm, achieving theoretical-level discovery and completion of physical equations and operators. These exhibit the highest explanatory power but also impose the most stringent requirements on data quality and computational resources.

The applicability of PIAI methods varies significantly across different urban subsystems. Each domain exhibits distinct characteristics in terms of data completeness, physical constraint strength, system openness, and dynamic coupling complexity. These factors determine the adaptation hierarchy and fusion strategy of various PIAI methods (M1–M7) in specific urban tasks. In general, Methods 1–3 (*Physics-Integrated AI*) are best suited for systems with well-defined conservation laws and stable topologies; Methods 4–5 (*Hybrid Ensemble*) are ideal for open, dynamic, and multi-source-driven systems; and Methods 6–7 (*AI-Integrated*

Physics) demonstrate unique advantages in systems lacking explicit governing equations or dominated by implicit social behaviors.

Energy. Energy networks-including power grids, district energy systems, and building energy consumption-feature explicit conservation and energy-balance equations with stable topologies and mature simulation models. They align most closely with M1, M2, and M3. M1 introduces power-flow or thermal conservation residuals into the loss function to preserve voltage and power balance with limited sensor data; M2 leverages simulation outputs from EnergyPlus or PSS/E for weight initialization, accelerating convergence and ensuring consistent thermal-electrical equilibrium; M3 employs graph or operator networks to represent multi-energy coupling topologies for unified energy-flow modeling. These methods fit energy systems well because of clear conservation equations and stable, simulatable structures. Future extensions may incorporate M4 to achieve “simulation-AI coupling,” enabling faster yet physically consistent power-flow and optimization solvers.

Environment. Urban air quality, meteorology, and hydrology represent open-boundary systems governed by explicit PDEs (e.g., advection-diffusion equations) but influenced by complex external drivers. They align best with M4 and M5. M4 uses physical solvers (WRF, CFD) to generate coarse-grained baseline solutions that neural networks refine locally; M5 runs physical PDE branches and neural ODE/Transformer modules in parallel-the former capturing transport processes, the latter learning external forcings and nonlinear disturbances. Sequential structures maintain numerical stability and spatial refinement, while parallel ensembles capture both interpretable and hidden dynamics. The main challenge lies in high-dimensional coupling among terrain, meteorology, and emission sources; incorporating M7 for turbulence or boundary-layer closure could enable multi-scale precision modeling.

Economy. Economic subsystems lack explicit physical equations but feature cross-sector coupling and time-lagged feedbacks that can be approximated as conservation-diffusion-feedback systems. They align best with M6 and M3. M6 uses sparse regression (SINDy [51]) to extract quasi-physical governing equations from temporal economic indicators, revealing dynamic relations among GDP, investment, and employment; M3 models industrial, regional, or trade networks using graph neural architectures embedded with input-output balance and feedback symmetry for interpretable economic forecasting. The combination of symbolic discovery and structured representation suits data-rich but mechanism-uncertain macroeconomic systems. Future research may integrate M5 to couple macro-level equations with micro-agent learning for unified multi-layer economic dynamics modeling.

Transportation. Urban traffic systems combine continuous conservation (e.g., LWR equation) with discrete agent interactions (e.g., IDM, social-force models), exhibiting strong nonlinearity and multi-agent effects. They align best with M3, M4, and M7. M3 captures flow conservation through graph or Hamiltonian neural networks; M4 refines macroscopic flow trends derived from fluid-dynamics solvers with AI-based residual corrections; M7 embeds learnable interaction terms into vehicle dynamics equations to achieve neural completion of real driving behavior. These combinations enable multi-layer fusion from network topology to individual-level modeling. Key challenges include behavioral heterogeneity and data-simulation alignment, which can be mitigated via differentiable simulation and causal modeling for improved generalization.

Information. Urban communication networks possess complex topologies and explicit physical-layer transmission laws (e.g., power, channel, and interference constraints). They align best with M3 and M5. M3 embeds power conservation and channel symmetry in graph structures for physically consistent network-state prediction; M5 jointly models physical transmission equations and traffic-prediction modules to support cross-layer QoS evaluation. These approaches balance modeling accuracy with real-time adaptability, suitable for wireless optimization and 6G network coordination. However, maintaining stable

differentiable physics under dynamic topologies remains challenging; a hybrid M4 can be introduced to form simulation-AI correction loops for fast optimization.

Public Services. Healthcare, education, and urban facility systems combine institutional constraints with population behaviors, featuring rich data but lacking unified governing equations. They align best with M5 and M6. M5 captures explicit resource-supply rules (e.g., scheduling, capacity) and nonlinear demand responses (e.g., mobility, preference shifts); M6 extracts implicit “demand-response-feedback” dynamics from historical service data, building symbolic, interpretable models. These approaches show strong potential in emergency care forecasting, education resource allocation, and facility usage modeling. The main challenge lies in susceptibility to policy shifts or disruptions, requiring dynamic symbolic updates and hybrid paradigm adaptation.

Emergency Management. These systems involve multi-physics coupling (e.g., fire, flood, wind) and human behavioral feedback, demanding extreme real-time performance and stability. They align best with M4, M5, and M7. M4 employs CFD or disaster simulators as the backbone while neural networks refine residuals or parameter tuning; M5 jointly models physical propagation and exogenous event response (e.g., meteorological or social-media signals); M7 embeds neural closure terms within PDEs (e.g., fuel diffusion, wind-pressure variation) to approximate real dynamics efficiently. This multi-level embedding framework achieves both interpretability and real-time responsiveness, proven effective in fire-smoke diffusion, urban flooding, and evacuation modeling. Data scarcity and high simulation cost remain challenges; future efforts may leverage generative augmentation and uncertainty quantification to enhance deployment robustness.

Urban Systems as a Whole. Comprehensive urban systems involve multi-domain coupling-energy, transportation, environment, economy, and information-characterized by cross-scale, multimodal, and multi-physics interactions. They align best with a hybrid combination of M4 and M5, supported by M3 and M7. The sequential framework models cross-domain backbone processes and global conservation trends, while parallel branches capture feedback loops among energy, emission, traffic, and economy. Unified architectural and operator embeddings enable multi-scale temporal-spatial forecasting, supporting digital-twin development and co-optimization of urban policies. Current bottlenecks include computational complexity and multi-source data alignment; future directions involve standardizing cross-domain interfaces and developing multimodal fusion operators for scalable integration.

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